VOLUME 21 JANUARY, 1933 NUMBER 1

PROCEEDINGS
of
The Institute of Radio Engineers

Institute of Radio Engineers Forthcoming Meetings

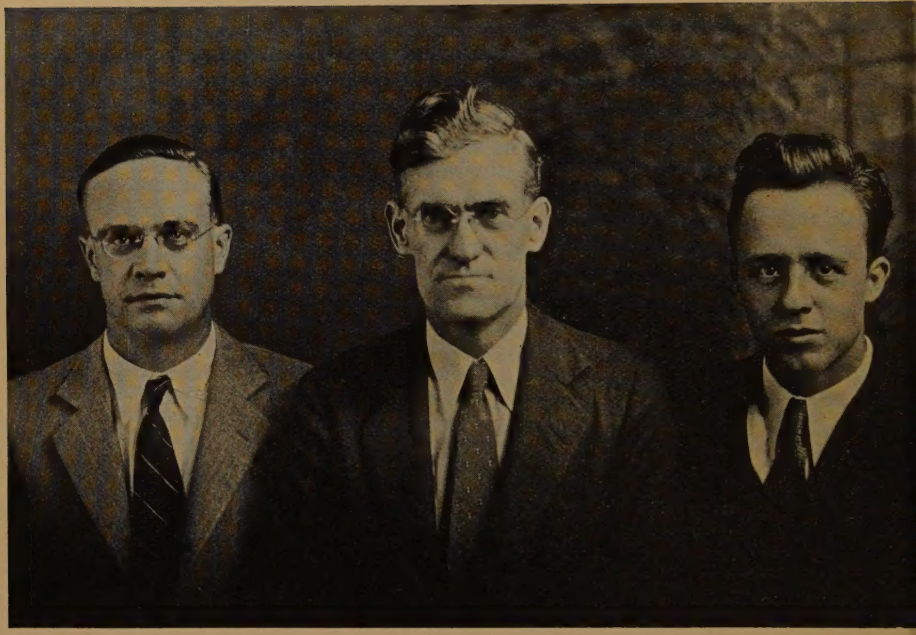
DETROIT SECTION
January 20, 1933

NEW YORK MEETING
February 1, 1933

PITTSBURGH SECTION
January 17, 1933

SAN FRANCISCO SECTION
January 18, 1933

WASHINGTON SECTION
January 12, 1933



OFFICERS OF THE CINCINNATI SECTION, 1932

Reading from left to right: W. C. Osterbrock, Vice Chairman; C. E. Kilgour, Chairman; H. G. Boyle, Secretary-Treasurer.

INSTITUTE NEWS AND RADIO NOTES

December Meeting of the Board of Directors

The December 7 meeting of the Board of Directors was held in the office of the Institute and was attended by President Cady, Melville Eastham, treasurer; R. H. Manson, Alfred N. Goldsmith, Arthur Batcheller, O. H. Caldwell, J. V. L. Hogan, H. W. Houck, L. M. Hull, C. M. Jansky, Jr., R. H. Marriott, E. L. Nelson, A. F. Van Dyck, William Wilson, and H. P. Westman, secretary.

Twenty-one applications for the Associate grade and four for the Student grade of membership were approved.

Approval was granted to a request for the holding of a Rochester Fall Meeting on November 13, 14, and 15, 1933.

Continuation of the affiliation of the Rochester Section of the Institute with the Rochester Engineering Society was approved.

A report of the Standards Committee, which is the result of the work of that body and its several technical committees during the past two years, was approved with a few minor items which were referred back to the committee for final consideration.

A proposed budget for 1933 was considered and with certain modifications was suggested to the new Board of Directors which will meet in January. In order to reduce expenditures during 1933, it was recommended to the new Board that no New York meeting of the Institute be held in June as the national convention will be held during that month. The possibility of dispensing with the September meeting was also considered, but no definite recommendation made.

No YEAR BOOK will be published in 1933. It was felt that changes are being made in the location of members at such a rapid rate as to make a catalog of membership, which is the basis of the YEAR BOOK, of little value in proportion to the substantial expense which its publication involves.

The Standards Report, which will be published early in 1933, will not be forwarded to all members as in the past but will be forwarded only to those members who request same. A slip to be returned to the secretary by those desiring copies of the Standards Report will be included in a future issue of the PROCEEDINGS.

Rochester Fall Meeting

The Rochester Fall Meeting, which was held on November 14 and 15, was attended by 188 of whom 110 were not residents of Rochester.

Those present represented sixty-even companies in the industry and nine educational or governmental institutions. They came from fourteen states and Canada.

The two-day meeting was devoted entirely to the presentation of the technical papers which were listed in the November PROCEEDINGS. It was felt that the meeting was extremely successful and maintained the standard which had been set up by the past Rochester Fall Meetings.

Radio Transmissions of Standard Frequencies

The Bureau of Standards transmits standard frequencies from its station WWV, Washington, D.C., every Tuesday. The transmissions are on 5000 kilocycles. Beginning October 1, the schedule was changed. The transmissions will be given continuously from 10 A.M. to 12 noon, and from 8:00 to 10:00 P.M., Eastern Standard Time. (From April to September, 1932, the schedule was from 2 to 4 P.M., and from 10 P.M. to midnight.) The service may be used by transmitting stations in adjusting their transmitters to exact frequency, and by the public in calibrating frequency standards, and transmitting and receiving apparatus. The transmissions can be heard and utilized by stations equipped for continuous-wave reception through the United States, although not with certainty in some places. The accuracy of the frequency is at all times better than one cycle (one in five million).

From the 5000 kilocycles any frequency may be checked by the method of harmonics. Information on how to receive and utilize the signals is given in a pamphlet obtainable on request addressed to the Bureau of Standards, Washington, D.C.

The transmissions consist mainly of continuous, unkeyed carrier frequency, giving a continuous whistle in the phones when received with an oscillatory receiving set. For the first five minutes there are transmitted the general call (CQ de WWV) and announcement of the frequency. The frequency and the call letters of the station (WWV) are given every ten minutes thereafter.

Supplementary experimental transmissions are made at other times. Some of these are made with modulated waves, at various modulation frequencies. Information regarding proposed supplementary transmissions is given by radio during the regular transmissions, and also announced in the newspapers.

The Bureau desires to receive reports on the transmissions, especially because radio transmission phenomena change with the season

of the year. The data desired are approximate field intensity, fading characteristics, and the suitability of the transmissions for frequency measurements. It is suggested that in reporting on intensities, the following designations be used where field intensity measurement apparatus is not used: (1) hardly perceptible, unreadable; (2) weak, readable now and then; (3) fairly good, readable with difficulty; (4) good, readable; (5) very good, perfectly readable. A statement as to whether fading is present or not is desired, and if so, its characteristics, such as time between peaks of signal intensity. Statements as to type of receiving set and type of antenna used are also desired. The Bureau would also appreciate reports on the use of the transmissions for purposes of frequency measurement or control.

All reports and letters regarding the transmissions should be addressed to the Bureau of Standards, Washington, D.C.

Proceedings Binders

Binders for the PROCEEDINGS, which may be used as permanent covers or for temporary transfer purposes, are available from the Institute office. These binders are of handsome Spanish grain fabrikoid, in blue and gold. Wire fasteners hold each copy in place, and permit removal of any issue from the binder in a few seconds. All issues lie flat when the binder is open. Each binder will accommodate a full year's supply of the PROCEEDINGS, and they are available at one dollar and seventy five cents ($1.75) each. Your name, or PROCEEDINGS volume number, will be stamped in gold for fifty cents (50c) additional.

Committee Work

BROADCAST COMMITTEE

A meeting of the Broadcast Committee was held at 7 P.M. on Tuesday, December 6, at the Institute office. Those present were E. L. Nelson, chairman; Arthur Batcheller, Q. A. Brackett, (representing H. F. Dart), J. V. L. Hogan, C. W. Horn, C. M. Jansky, Jr., L. F. Jones (representing B. R. Cummings), R. H. Marriott, and H. P. Westman, secretary.

The committee agreed to prepare an analysis of the present broadcast situation as it concerns the relative number of clear channels and regional channels, pointing out the advantages and limitations of both types of services. The preparation of such an analysis was started, and some tentative drafts discussed in detail.

Some proposed performance requirements for broadcast transmitters were submitted and distributed to the members for further action at the next meeting.

It was felt desirable to include in the list of matters to be studied the subject of the geographical placement of high power broadcast stations in relation to the densely populated areas they are intended to serve.

Membership Committee

The Membership Committee meeting, scheduled for 5:30 P.M. on December 7 at the Institute office, was attended by H. C. Gawler, chairman, and A. M. Trogner who prepared a statement covering the activities of the committee during the past year.

Institute Meetings

Buffalo-Niagara Section

A meeting of the Buffalo-Niagara Section, held on November 23 at the University of Buffalo and presided over by V. C. MacNabb, was attended by forty members and guests.

The evening was devoted to a paper on "The Manufacture and Use of Carbon Resistors," which was presented by L. M. Perkins, chief engineer of the Erie Resistor Corporation.

Mr. Perkins presented a brief history of the development of resistors used for radio receiving purposes and described the construction of wire wound resistors, film or filament resistors, such as those made from ink and pencil marks, and molded composition resistors made by baking or casting a mixture of finely divided particles of resistance material, filler, and binder. Engineering and manufacturing advantages and limitations of the different types were given. It was pointed out that the resistance of carbon type units resided chiefly in the contacts between particles of carbon rather than within the particles. The variation of resistance due to age, variations in current and voltage, frequency, moisture, and temperature was covered.

A number of these characteristics were shown by graphs, and a cathode ray oscillograph was employed to illustrate these effects upon various types of resistors. At the close of the presentation, a general discussion was held.

Chicago Section

The Chicago Section held a meeting on October 14 in the meeting room of the Western Society of Engineers which was presided over by R. M. Arnold, vice chairman.

K. W. Jarvis of the Zenith Radio Corporation, presented a paper on "Intercarrier Noise Suppression Circuits in Radio Receiving Sets."

The introductory portion of the paper pointed out that the increased sensitivity of modern radio broadcast receivers employing automatic volume control resulted in their appearing to be noisy, especially when tuned between station carriers. The possibilities of using mute switches requiring a visual method of indicating resonance and methods automatically disconnecting the audio system when the tuning knob is operated were outlined.

The author then discussed the use of automatic silencing of the audio system through circuits controlled by the automatic volume control tube. Several such arrangements were shown, each based on amplitude of signal input. A second group of methods was illustrated in which a cut-off of the audio channel was based on a specific degree of detuning from the carrier. The functions of these two groups, amplitude control and frequency control, were then combined in a third group of circuit arrangements. Indication was made of the possible usages of relays and thyratron tubes as well as the more conventional method of overbias. Certain limitations regarding fading, criticalness of tuning, and general acceptibility of performance closed the paper which was discussed by Messrs. Arnold, Hucks, Vance and others of the 125 members and guests in attendance.

Cincinnati Section

At the University of Cincinnati on November 22 was held a meeting of the Cincinnati Section. C. E. Kilgour, chairman, presided.

Before introducing the speaker of the evening, the chairman presented a short résumé of the Rochester Fall Meeting and the papers presented at it.

J. M. Glessner of the Crosley Radio Corporation then presented a paper on "Some Further Notes on Diode Detectors" which had been delivered by the coauthor, Mr. Kilgour, at the Rochester Fall Meeting. The paper was discussed by Messrs. Felix, Israel, and Osterbrock.

Thirty members were present at this meeting.

Detroit Section

H. L. Byerlay presided at a joint meeting of the Detroit Section and the Detroit Engineering Society which was held in the Detroit Engineering Society auditorium on November 18. The attendance totaled 110.

"Engineering Radio Receivers for the User" was the subject of a

paper by Virgil M. Graham of the Stromberg-Carlson Telephone Manufacturing Company.

The speaker outlined a number of useful operating characteristics which should be possessed by an ideal receiver for broadcast reception, and pointed out the difficulties which are many times encountered in specifying these requirements. A number of undesirable characteristics which are many times found in receivers were described together with methods employed in their elimination or reduction. A general discussion followed the paper.

Los Angeles Section

E. H. Schreiber presided at the November 15 meeting of the Los Angeles Section held at the Mayfair Hotel.

The subject of the meeting was "The Technical Application of Vacuum Tubes," and papers were presented by C. R. Daily of Electrical Research Products, Inc., and J. F. Blackburn, consulting physicist. Dr. Daily described the development of the Western Electric 262-A tube for alternating-current operation for high gain speech amplifier equipment. He outlined the advantages of high-voltage low-current filaments as contrasted with low-voltage high-current operation. Isolation of the grid lead from the filament and plate leads and shielding of the cathode were pointed out as being important factors in quiet operation of this tube.

The second paper by Dr. Blackburn covered several vacuum tubes not commonly used in communication work. These included cold and hot cathode tubes, hot cathode mercury vapor rectifiers, voltage regulating tubes, and thyratrons. Data were supplied for some of the tubes described, and a cathode ray and a thyratron tube were on display.

Fifty-five members and guests attended the meeting, seventeen of which were present at the informal dinner which preceded it.

New York Meeting

The regular New York meeting was held on December 7 in the Engineering Societies building and was presided over by President Cady.

F. M. Ryan of the Bell Telephone Laboratories presented a paper on "Some Recent Advances in Radiotelephone Equipment for Transport Airplanes."

The author pointed out that commercial air transportation in the United States has experienced a very rapid expansion during the past five years. In this era of expansion, an important rôle has been played

by radiotelephony, which, by providing contact between transport airplanes and ground, has contributed much to safety and reliability.

To keep step with the advance in air transportation, radio practice has undergone changes as great as those which have taken place in the field of aeronautics during the same period. The result has been the production of highly developed radio equipment especially adapted to the needs of air transportation, and differing radically in design from conventional types of radio apparatus.

The equipment described was the most recent step in a continuing development program which has resulted in the design of new two-way radiotelephone apparatus for transport airplanes embodying many unusual features such as rapid frequency-change facilities, crystal control of both transmitter and receiver, and automatic gain control.

The meeting was attended by 250 members and guests, a number of whom participated in the discussion which followed the presentation of the paper.

Philadelphia Section

The Philadelphia Section held its November meeting on the 3rd at the Engineers Club. The meeting was presided over by Chairman H. W. Byler.

Two papers were presented, the first on "Studio and Remote Speech Equipment," by W. L. Lyndon of the RCA Victor Company, and the second on "Fifty-Kilowatt Installation at WCAU in Philadelphia," by J. E. Love of the RCA Victor Company.

Mr. Lyndon's paper dealt with the studio and speech equipment installed in the new studio building of WCAU in Philadelphia. He was followed by Mr. Love who described the new fifty-kilowatt transmitter located at Newtown Square. He also described briefly the new short-wave transmitter W3XAU which has recently been installed.

Following the presentation of the papers, J. G. Leitch, Technical Director of WCAU, extended an informal invitation to those present to visit the transmitter and the meeting was adjourned to Newtown Square. The Philadelphia Police Department coöperated in the handling of the problem of convoying the caravan of ninety-three automobiles which were on hand to transport the members to the station. Parking regulations at the Engineers Club were suspended and a police escort was on hand to facilitate the movement of this large group of cars through the city. Upon arrival at Newtown Square, Mr. Leitch conducted the party and groups to the station which was housed in a building of modernistic design containing every facility required by a modern transmitter. The section is indebted to the owner and personnel of WCAU for making possible this enjoyable and instructive visit.

Three hundred and four members and guests were present.

The Philadelphia Section has announced its program of meetings for the first five months of 1933. These are given below:

January 5, 1933
"Aircraft Radio, Its Growth and Future," by Harry Diamond of the U. S. Bureau of Standards.
February 2, 1933
"Electric Carrillon," by E. B. Paterson of RCA Victor.
March 2, 1933
"RCA Communications Network"
April 6, 1933
"University Night"
May 4, 1933
"Short-Wave Directional Antennas"

Pittsburgh Section

A meeting of the Pittsburgh Section was held on November 22 at the University of Pittsburgh. It was presided over by Chairman R. T. Griffith, and the attendance totaled eighty-five.

A. E. Ruark of the Physics Department of the University of Pittsburgh presented a paper on "Modern Methods for the Study of Radioactive and Cosmic Rays."

Dr. Ruark outlined the history of the various methods used in the detection of individual particles and rays emitted by radioactive substances. Starting with the Wilson cloud chamber, the speaker interestingly described the advancement made in this type of apparatus, until at the present time radio technique and vacuum tube circuits are being used to supplement these earlier methods.

The speaker described in detail the theory and circuits of vacuum tube electroscopes and electrometers illustrating with models and slides the ingenious tubes and circuits developed for use in conjunction with various counters for the detection of the beta and gamma rays or cosmic ray effects.

After the adjournment of the meeting, a number of the laboratories in which extensive research equipment was in operation were opened for general inspection.

San Francisco Section

The November meeting of the San Francisco Section was held jointly with the local section of the American Institute of Electrical Engineers at the University of California. The meeting was presided over by the A.I.E.E. section chairman, E. F. Maryatt.

"Experimental Methods of Atomic Disintegration" was the subject of a paper by O. E. Lawrence of the Physics Department of the Univer-

sity of California who was introduced by Dr. Fuller of the University of California who is also a member of both the I.R.E. and the A.I.E.E.

Dr. Lawrence in his well-illustrated lecture discussed several methods used at the University of California for the production of high speed ions without the use of high voltages. A description was given of the recent experiments by means of which lithium has been disintegrated into helium, utilizing bombardment with high speed ions. A demonstration in the Radiation Laboratory followed the lecture, and it was shown that by the use of a very strong magnetic field, ions could be produced having a velocity of motion equal to that which would be imparted to them by a potential of 4,200,000 volts. A new and extremely powerful device for the production of X-rays was also shown.

The meeting was attended by 350 members and guests of both societies.

Washington Section

On November 10 a meeting of the Washington Section was held at the Kennedy-Warren Apartment Hotel. H. G. Dorsey, chairman of the section, presided, and the attendance totaled sixty.

The paper of the evening "Cathode Ray Tubes—Their Characteristics and Applications" was presented by A. B. DuMont. This paper appeared in the December, 1932, issue of the Proceedings.

Personal Mention

Lieutenant G. J. Crosby, U.S.N., has been transferred from the U.S.S. Memphis to Balboa, C. Z.

Formerly with the Delco-Remy Corporation, H. C. Forbes has joined the engineering staff of the U.S. Radio and Television Corporation, Marion, Ind.

J. F. Blackburn, previously at the California Institute of Technology, has established a practice as a consulting physicist.

Formerly with RCA Victor Company, Peter Caporale is now consulting engineer for the Electro-Acoustical Engineering Company of America in Philadelphia.

E. F. Carter, formerly with United Research Corporation, is now doing consulting work with headquarters at Laurelton, L. I., N. Y.

Lieutenant L. R. Daspit, U.S.N., has been transferred from the U.S.S. Childs to the Submarine Base at New London, Conn.

Previously with Wired Radio, R. J. Davis has joined the engineering department of the DeForest Radio Company of Passaic, N. J.

F. H. Engel has left the Hygrade Lamp Company to join the Engineering staff of the RCA-Radiotron Company at Harrison, N. J.

Formerly with Les Laboratories Standard of Paris, W. T. Gibson has become chief valve engineer for Standard Telephones and Cables, London, England.

Major G. W. Graham, U.S.A., has been transferred from Savannah, Ill., to the Army War College at Washington, D.C.

Previously with Gray and Danielson Manufacturing Company, A. H. Hart has joined the staff of Mackay Radio and Telegraph Company, San Francisco, Calif.

A. H. Hotop is now with the DeForest Radio Company of Passaic, formerly being connected with Wired Radio.

Previously with the United Air Cleaner Corporation, H. J. Kayner has joined R. H. G. Mathews and Associates of Chicago, Ill.

Formerly with the National Sound Service Bureau, H. J. Kempf is now connected with the Ashland Laboratories of Chicago.

M. W. Kenney has left the Allen-Bradley Company to become chief engineer of the Grunow Corporation of Berwyn, Ill.

H. S. Knowles has been advanced to chief engineer of the Jensen Manufacturing Company.

C. J. LeBel is now doing consulting work having previously been connected with the Hygrade Sylvania Corporation.

Formerly with Bell Telephone Laboratories, H. W. Lederhaus has established the firm of H. W. Lederhaus and Company at Jackson Heights, L. I., N. Y.

Previously with Electrical Research Products, Nathan Levinson, has become director of recording for Warner Brothers-First National Studios at Burbank, Calif.

W. A. McCutcheon formerly with the Western Electric Company in Sydney, Australia, is now with Electrical Research Products in Buffalo, N. Y.

Previously with Les Laboratories Standard of Paris, D. B. Mirk has become a radio engineer for Standard Telephones and Cables in Hendon, England.

H. B. Nielson has joined the engineering department of the U.S. Radio and Television Company of Marion, Ind., formerly being with the Brunswick-Balke-Collender Company.

Previously with Transcontinental Western Air, T. E. Nikirk has become an instructor for the National Radio and Electrical School of Los Angeles, Calif.

E. E. Parisek previously with P. R. Mallory and Company has become chief engineer of Premier Electric Company of Chicago.

Formerly with Atwater-Kent Manufacturing Company, S. W. Place is now connected with the Synthane Corporation of Oakes, Pa.

J. H. Pressley has been advanced to the vice presidency of the U. S. Radio and Television Corporation.

Formerly with the Television Manufacturing Company of America, Morris Rappaport has joined the staff of the Electronic Engineering Corporation, Long Island City, N. Y.

L. B. Root formerly with General Amplifier Company has become an instructor in the Samuel Curtis Radio School of Boston, Mass.

F. J. Smith, inspector for the Federal Radio Commission, has been transferred from Detroit, Mich., to Buffalo, N. Y.

Captain J. A. Stansell, U.S.A., has been transferred from New Haven, Conn., to Fort Monmouth, N. J.

A. W. Steinberger is now with the General Instrument Corporation of New York City having previously been on the staff of the Rudolph Wurlitzer Manufacturing Company.

Lieutenant T. T. Teague, U.S.A., is now on the faculty of the Signal School at Fort Monmouth, N. J. He was formerly in the telegraph service at Ketchikan, Alaska.

Previously with International Standard Electric Corporation, A. E. Thompson has become technical director of Creed and Company, Lmited, of Croydon, England.

W. H. Troutbeck has left Standard Telephones and Cables to become technical director of Centralized Sound Apparatus Company of London, England.

Proceedings of the Institute of Radio Engineers
Volume 21, Number 1 *January, 1933*

TECHNICAL PAPERS

AN OSCILLATOR HAVING A LINEAR OPERATING CHARACTERISTIC*

By

L. B. Arguimbau

(General Radio Company, Cambridge, Mass.)

Summary—*A review of conventional linear equilibrium conditions is given. It is shown that these conditions are not usually at all applicable to practical oscillators because the operating region is not to be so simply described. The relation of nonlinear effects to frequency modulation is pointed out. A modified type of oscillator which conforms to the elementary linear conditions is described and its properties are discussed. The results are applied to a conventional grid leak and condenser oscillator to explain its modulating characteristics. The simplicity of the new circuit permits a discussion of amplitude inertia effects.*

ANY attempt to obtain an equilibrium condition for the usual type of oscillating vacuum tube circuit leads to a rather imposing set of equations. Reduced to its lowest terms, such a problem really requires the solution of a system of simultaneous linear

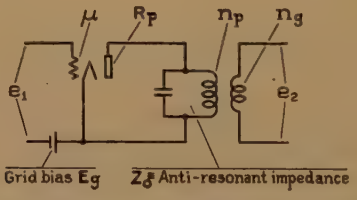


Fig. 1—Tuned amplifier circuit. For simplicity in this and in the following figures, the plate battery is omitted.

differential equations connected with the fixed circuit elements, together with the empirical equations for the tube itself. Even worse than this, to satisfy one's mathematical scruples, it might be necessary to investigate rather elaborate boundary conditions, also the uniqueness of the solution. This program is usually too ambitious to be at all useful.

One of the commonest methods of evading the difficulty consists in arguing by analogy to the ordinary linear electric circuit theory, re-

* Decimal Classification: R133. Original manuscript received by the Institute, July 5, 1932. Presented before U.R.S.I., Washington, D. C., May 1, 1931.

placing the plate circuit of the tube by its so-called internal resistance. In the first part of the present paper, it is planned to do this, pointing out that the method really dodges the main issue of establishing equilibrium. After this review, a simplified oscillator circuit will be exhibited in which the simple analogy is truly applicable. Later the methods developed in this ideal case will be applied to explain the operation of the grid leak and condenser oscillator.

I. Review of Equilibrium Conditions for Conventional Oscillator[1]

Fig. 1 shows a typical tuned amplifier circuit. It will be shown that the linear equilibrium conditions for an oscillator such as that shown in Fig. 2 can be very readily derived from the amplifier circuit. In all

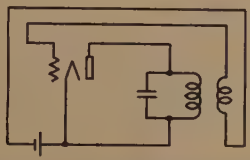


Fig. 2—The circuit of Fig. 1 has been connected as an oscillator.

of the following, unless otherwise stated, it will be assumed that the conductive grid impedance is infinite. (In the circuits to be discussed under Parts 2 and 4, this is realized since normally no grid current flows.)

If the input is very small, the voltage amplification of the amplifier can be written down quite readily.[2] For a given tube and load circuit, the gain will depend upon the plate resistance and hence upon the grid and plate biases; in the present case it will prove most convenient to consider the gain as a function of the grid bias alone, the other factors being held constant. If a suitable bias is chosen and the resonant frequency is applied, the output e_2 will be exactly equal to the input e_1 in both phase and amplitude. When this condition has been obtained it might appear possible, as in Fig. 2, to disconnect the generator from the input, replacing it by the amplifier output. Since no change is apparent, the circuit might be expected to function as before. In practice,

[1] *See* J. W. Horton, "Vacuum tube oscillators—a graphical method of analysis," *Bell Sys. Tech. Jour.* vol. III, pp. 508–524; January, (1924), for a more general treatment.

[2]

$$e_2/e_1 = \frac{\mu \frac{N_g}{N_p}}{1+\frac{R_p}{Z_0}}$$

however, it is very difficult, if not impossible, to realize these conditions. Unless extreme precautions are taken the circuit either ceases to function or builds up very violently in amplitude until the tube overloads; i.e., over the operating region the plate current does not vary at all linearly with plate and grid voltages.

The difficulty lies in the type of equilibrium stability. It usually happens that the effective resistance of the tube at low signal intensities varies very slowly with amplitude. This is because the change in

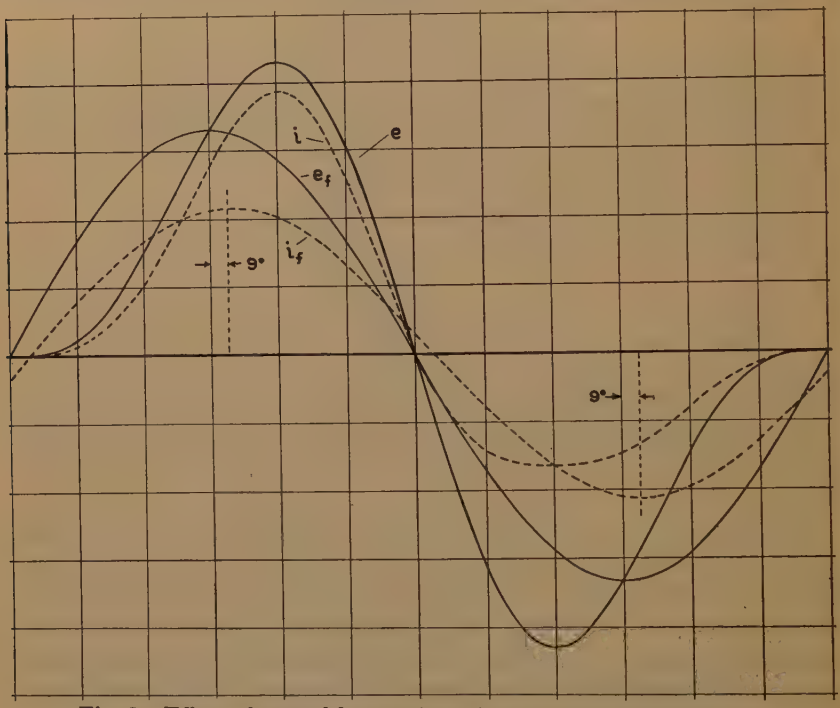


Fig. 3—Effect of second harmonic on impedance to fundamental.

effective resistance depends only on 3rd, 5th, and higher odd-order terms in a series expansion, not upon the lower terms. In case the resistance increases with amplitude, equilibrium can be established at low intensities, but adjustments become exceedingly critical. In the other case where the effective resistance decreases with amplitude, it is not possible to establish a stable equilibrium state at small amplitudes; the tube must overload.

These critical equilibrium conditions which usually mean that an oscillating vacuum tube is completely nonlinear, make the operation somewhat complicated. Even if grid current is neglected it is neces-

sary to discard the picture of a fixed equivalent plate resistance defined by partial derivatives. In fact, it is plain that the impedance used in the above derivation should be defined as the vector ratio of the fundamental components of voltage and current. (In the case of a three-element vacuum tube, the definition must be modified to take into account the equivalent plate-circuit voltage corresponding to the grid swing.) When the plate voltage and current wave form are badly distorted (in practice the plate current often drops to zero for an appreciable portion of the cycle), this somewhat artificial definition has little connection with the static constants of the tube.

This whole problem has been outlined in a paper by Dr. Peterson of the Bell Telephone Laboratories.[3] A physical picture of one of the difficulties can be obtained from a careful study of Fig. 3. Due to the reactance offered by the resonant circuit to the second harmonic, a voltage wave of the type shown by curve e might be expected. The curve e_f shows the fundamental component of the wave. Impressing the first curve on a parabolic characteristic the wave i is obtained, the fundamental component being given by the dotted curve. It will be noticed that there has been a shift in phase between the fundamental components of voltage and current. A consideration of these curves from the viewpoint of the integrating method of evaluating Fourier's coefficients gives a rather clear picture of the process.

Phase shifts of this nature in the case of an oscillator must be compensated for by corresponding shifts in the frequency, shifts enabling the tuned circuit to operate slightly off resonance. Since the amount of the phase shift will depend on the amount of distortion and hence upon the grid and plate voltages, one cause of frequency modulaton becomes quite apparent. In case the coupling coefficient between the windings of the oscillator coil is small, the impedance offered to the 2nd harmonic by the leakage reactance may be much larger than the reactance of the condenser, and thus increase frequency modulation.

II. An Oscillator Having a Linear Operating Characteristic

It has been seen in the above that the essential difficulty in explaining the action of the usual type of oscillator lies in the fact that no noncritical stable equilibrium can be established unless the tube is permitted to operate over a thoroughly nonlinear region, a condition which is incompatible with ease of computation, and introduces the effects which have been reviewed. If it were possible to set up an oscil-

[3] Eugene Peterson, "Impedance of a non-linear circuit element," *Trans. A.I.E.E.*, vol. 46, pp. 528–534; May, (1927). Much of the present discussion concerning phase shifts is based on the ideas in this paper.

lator whose damping increased rapidly with increases in amplitude, preferably linearly at low intensities, the difficulties would be over for everything would be readily calculable, and the tube would function linearly without the distortional troubles.

An oscillator realizing these conditions is shown schematically in Fig. 4. It will be noticed that the sole change is the substitution of an amplitude controlled bias for the C battery. (By the use of a crest-type rectifier,[4] it is quite easy to obtain a steady voltage which is accurately proportional to the applied swing.) For the present purposes such a system may be described as an oscillator whose bias may be chosen as any constant multiple of the grid or plate oscillating voltage.

In order for the tube to oscillate without appreciable distortion, the grid bias must correspond to the equilibrium value of plate resistance required by the simple linear theory. If the oscillator were to be started artificially with a low amplitude, the bias provided by the recti-

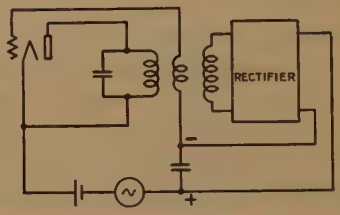


Fig. 4—A circuit with grid bias a function of amplitude of oscillations.

fier would be small, perhaps smaller than the equilibrium value. In this case, the equations of the circuit would indicate an exponentially increasing solution; the amplitude would increase as long as the bias corresponded to a lower resistance than the equilibrium value. On the other hand, if the amplitude were increased above this value, artificially, the increased rectifier output would give rise to a plate resistance higher than the value for equilibrium and as a result the oscillations would be damped down to the previous value.

An apology may be necessary for the following, but the quickest and perhaps the clearest way of describing the action of the circuit can be had in the discussion of a typical numerical example. Consider the circuit shown in Fig. 4. Here, as a typical case, it may be assumed that a vacuum tube with an amplification constant of 8, operating with a fixed plate battery of 180 volts, is connected with a tuned transformer having a plate-to-grid turns ratio of 4:1. An antiresonant impedance of 7500 ohms is assumed for the plate coil. A simple calculation will

[4] *See* e.g. C. H. Sharp and E. D. Doyle, "Crest voltmeters," *Trans. A.I.E.E.*, vol. 35, pp. 99–107; February, (1916).

show that in this particular case the plate resistance should be equal to the tuned circuit value, 7500 ohms. The figures given correspond approximately to those for a type 112-A tube.

(This can be very readily seen as follows: Start from the plate winding. A given voltage here will correspond to a grid swing of one-fourth of this value. Due to the amplification factor of 8, this will appear as twice the plate swing in series with the internal impedance. This in turn must be halved by the plate load so that the original voltage is obtained again; in other words, the internal resistance must be equal to the plate load of 7500 ohms.)

From the static characteristics of the tube, it turns out that this plate resistance of 7500 ohms can be obtained by a bias of -15 volts on the grid. If no biasing battery is used, this means that the output of the rectifier must be -15 volts. If, for simplicity, it is assumed that the coil supplying the rectifier is arranged to make the rectified output equal to the peak plate swing, this swing must have an amplitude of 15 volts.

To sum up: For equilibrium it was found necessary to have a bias of -15 volts on the grid. This bias in the present case corresponds to a plate swing of 15 volts so that equilibrium can be established with this one particular amplitude; any other amplitude would give a transient solution, and the amplitude would be returned to the stable value automatically.

What would be expected to happen if this equilibrium were disturbed by the arbitrary insertion of an additional bias by means of a battery, say of -10 volts? The total voltage here, -25 volts, would correspond to a plate resistance higher than 7500 ohms, and the amplitude would be damped until the net bias returned to the equilibrium value of -15 volts; in other words, until the output of the rectifier was -5 volts; that is, until the plate swing was 5 volts. It will be noticed that a change in grid bias has given rise to a corresponding change in plate swing.

Suppose that, instead of using a biasing battery of -10 volts, a positive bias of, say, $+15$ volts were inserted. Here, for equilibrium, the rectifier would have to produce a voltage of -30 volts, corresponding to a plate amplitude of 30 volts. Under these conditions, it will be noticed that the peak grid swing (which is one-quarter of the plate swing by hypothesis) would be only 7.5-volt, a value which would be numerically less than the -15-volt bias, so that no grid current will be drawn.

The next obvious step in the argument consists in replacing the biasing battery by a slowly varying sinusoidal signal. Clearly, the am-

plitude of oscillation must follow this signal linearly. In other words, a completely linear oscillator-modulator system has been obtained. If, in particular, the modulating signal had a peak swing of 15 volts, the output signal would be linearly modulated up to 100 per cent. It may be worth pointing out that the fractional modulation is given directly as the ratio of the peak modulating signal to the rectifier output, a fact which has been found convenient in circuits for generating standard modulated signals.

It will be noticed that at no point of this argument has it been required that the tube should overload. In fact, it is quite easy to make it operate over a linear region of its characteristics. This means that there is very little distortion, and due to this freedom from distortion, the phase shifts previously mentioned are very small, and hence the related frequency modulation is low.

III. Explanation of the Conventional Grid-Leak and Condenser Type of Oscillator

The operation of a grid-leak condenser oscillator is somewhat similar to that of the type just described because here again the grid bias is a function of the amplitude of oscillation. The essential difference between the two types consists in the nature of this functional relationship. The linear approximations made in the case treated above do not hold nearly as accurately with a condenser-leak circuit because in the first case the bias produced by the grid current is not nearly as large as that which can be obtained in the other case. It will be shown later that for a close linear approximation the ratio of rectified bias-to-plate swing should be as large as possible. In many cases the approximation is so poor that little similarity can be recognized, and other explanations are needed. In a grid-leak condenser oscillator, where a very high resistance leak is used and a condenser which offers a negligible impedance to the oscillator frequency, the grid current must be zero because no appreciable current can flow through a resistance which can be assumed infinitely large. This condition means essentially that the maximum positive voltage applied to the grid must always be equal exactly to zero, or rather, to the grid current cut-off voltage. Thus, the algebraic sum of the effective grid bias and the numerical peak swing must be equal to the grid current cut-off voltage.

On the other hand, in order to obtain linear equilibrium it has been seen that for a given plate voltage the bias must have a definite value. If in the previous numerical example a grid leak and condenser had been used in place of the external rectifier, a bias of -15 volts would have been required. Whether this bias was obtained by the grid-current

drop or by a battery, the condition that the maximum positive grid voltage must be equal to the cut-off value would necessitate a peak oscillatory swing of 15 volts. *This means that the swing is to a first approximation independent of grid bias as long as the grid can swing positive.* If the bias were more negative than −15 volts, the oscillator would stop oscillating. Fig. 5a is an experimentally determined charac-

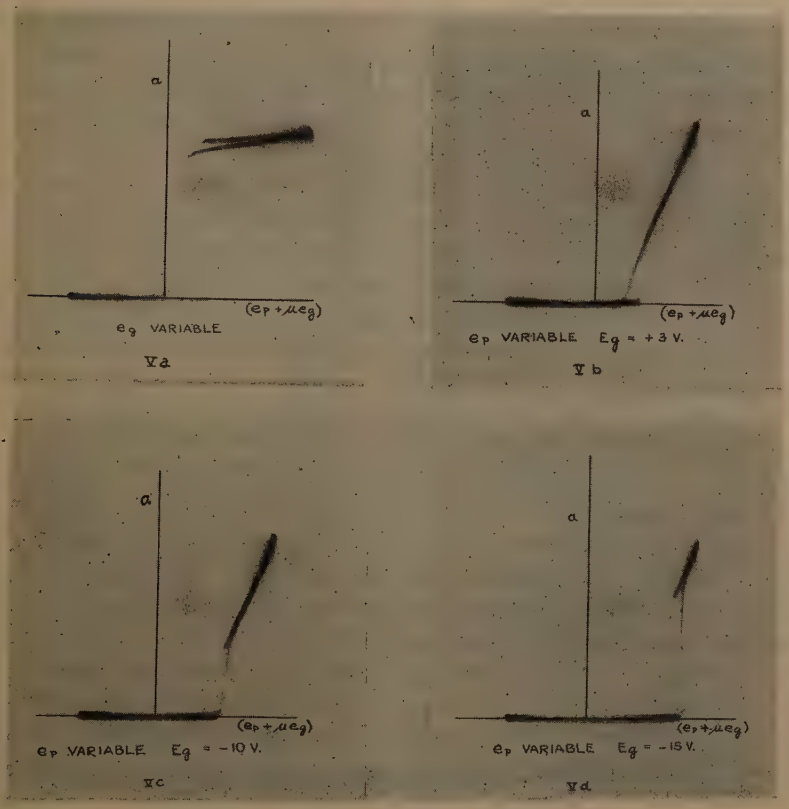


Fig. 5—Experimental modulation characteristics for grid leak and condenser oscillator. (Note discontinuities due to I_g dropping to zero.)

teristic of this sort. The curve was taken by applying the low-frequency modulating voltage to the horizontal plates of a cathode ray oscillograph, and connecting the vertical plates to the output of a linear rectifier shunted across the tuned circuit. Except for transients at the cut-off point, the curve follows the argument given. The small slope of the curve at the right is mainly due to the fact that the grid circuit does not act as the ideal rectifier which was assumed.

If an attempt is made to modulate the oscillator in the plate circuit, the results are quite different. Here the necessary equilibrium grid bias varies linearly with the plate voltage. Since the grid swing must be equal to the necessary grid bias, this means that the amplitude will vary linearly with the plate voltage. On the other hand, if the plate voltage is decreased until the value of $e_p+\mu e_g$ (where e_g is now the part of the bias due to a battery) reaches the equilibrium value, all of the bias due to grid current must have dropped to zero. In other words, at this point the peak grid swing has dropped to the value of the grid biasing battery. Further decreases in plate voltage cannot be compensated for, and the circuit ceases to oscillate, causing a discontinuity in the modulation characteristics.

Fig. 5b, c, and d show the curves obtained by plate modulation of this sort. The abscissas have been decreased by a factor of μ so that these curves are plotted to the same scale as that of Fig. 5a for grid modulation when the amplitude is considered as a function of $(e_p+\mu e_g)$. It will be noticed, as might be expected, that the slope is of a different order of magnitude. Once more the transients are noticeable near the discontinuity.

IV. Extension of Simple Theory to Include Nonlinear Correction and Dynamic Effects

There are several factors in the study of oscillators which are usually annoyingly complicated, but which can be treated with relative ease under the present conditions. In the first place, frequency modulation is more readily explained, since many disturbing elements have been removed. A physical picture of this process has been given above, but an attempt to put this in mathematical form leads to rather complicated results, and so it has been omitted from the discussion.

A second point which should be mentioned is the departure of the modulation characteristic from linearity due to curvature in the tube characteristics. In the present case it has been shown that neglecting relatively small corrections, (which as yet haven't even been mentioned) the amplitude varies linearly with plate and grid biases, even up to the point where oscillations cease. As a matter of fact, the effective plate resistance varies somewhat with amplitude. To compensate for this change in resistance, the effective bias necessary for equilibrium in one of the "linear" oscillators will vary slightly with amplitude. A simple calculation will show that if the amplification factor is assumed constant, and a constant ratio, n_g/n_p is maintained between the plate and grid swings, the change in resistance will be a function of $[(n_g/n_p)\mu-1]\,e_p$. Again it should be noted that this change of resistance de-

pends only on odd-order coefficients in the series expansion so that it is independent of the square term. For any particular ratio, n_g/n_p the change in resistance due to a change in plate swing from A to B must be compensated by a shift in grid voltage of, say, E volts. If the ratio of rectifier output to plate swing is β, this will mean a departure from plate output linearity of E/β. It follows that the higher the ratio of rectifier output to plate swing, the more closely will the linear approximations hold.

It is quite easy and instructive to substitute numerical estimates or experimental values for the nonlinear compensation values and note how almost any degree of linearity can be obtained by proper design. The smallness of the nonlinear correction and the control over it obtained by the plate-to-rectifier ratio should be borne in mind in connection with the discussion of the conventional grid-leak and condenser oscillator which has been given above. In connection with this point it might be added that the amplitude will not change nearly as rapidly with resonant impedance as in the case of the usual oscillator. This accounts for the fact that with a properly proportioned linear oscillator the tuning capacity can be changed by 10:1 with a change of only about 2 per cent in amplitude.

A third point in which the linear oscillator adapts itself to computation is in the treatment of dynamical modulation, or so-called "flywheel" effects. These effects are due essentially to the fact that a tuned circuit does not build up to its full amplitude instantaneously when connected to a source. When the amplitude is plotted as a function of the modulating voltage over the audio-frequency cycle instead of a straight line a sort of hysteresis loop results. The arithmetic is rather cumbersome so that only the essential points will be sketched below.

In the case of the linear oscillator it will be desirable to make use of the physical concept of the instantaneous amplitude of an oscillatory function. (A closely analogous situation is pointed out and treated at length by Planck.)[5] The instantaneous amplitude may be described as the maximum value during any one high-frequency cycle of the oscillatory function: it is assumed that the fractional change in the value of this maximum from one cycle to the next is very small.

Once the concept has been granted the rectifier output can be considered as a function of this instantaneous amplitude, thereby relating the damping to the amplitude. The introduction of the idea of instantaneous amplitude, a quantity which cannot be conveniently expressed

[5] Max Planck, "Wärmestrahlung," 5, chapter 1, section 3, pp. 3–4, J. A. Barth, Leipzig, (1923).

analytically, would be expected to give rise to difficulties, and this is true here. The usual differential equation methods are not directly applicable to the case of a tuned circuit whose damping is a function of the amplitude, and it appears more convenient to argue on an energy basis.

Consider the tuned circuit of Fig. 6. The energy stored by the inductance is $\frac{1}{2}Li_L^2$; that stored by the condenser is $\frac{1}{2}Ce^2$. Let a be the "instantaneous high-frequency voltage amplitude;" i.e., the maximum voltage across the condenser C during the high-frequency cycle considered. At the time of maximum voltage the condenser current will be

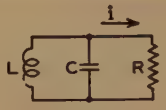


Fig. 6

$$R = \frac{Z_0}{1 - \alpha \frac{Z_0}{R_p}}$$

$$\alpha \equiv \frac{n_g}{n_p}\mu - 1$$

zero so that the inductance current will be equal to that through the resistance or a/R making the energy stored by the inductance $\frac{1}{2}L(a^2/R^2)$ giving a total energy stored by the resonant circuit of $\frac{1}{2}(C+L/R^2)a^2$. The energy stored by the circuit at the next voltage maximum will be approximately

$$W + \Delta W = \left[\frac{1}{2}\left(C + \frac{L}{R^2}\right)a^2 - \frac{a^2}{2\overline{R}}\Delta t\right] \tag{1}$$

where $\overline{R}$ is a mean of R during the cycle. (This statement is essentially only an assumption but a highly probable one.) This expression suggests the equation,

$$\frac{da}{dt} = \frac{-a}{2(CR + L)} \tag{2}$$

or approximately,

$$\frac{da}{dt} = \frac{-a}{2CR}.$$

It will be more convenient to write this in the form

$$\frac{da}{dt} = \frac{-pZ_0}{2Q}\,\frac{a}{R} \tag{3}$$

where,

p is the angular frequency of the oscillations,

Z_0 is the impedance of the antiresonant circuit, and Q is the X/R ratio of the tuned inductance.

This result describes the action of a tuned circuit with slowly varying damping. The next step consists in applying this to the oscillator circuit. It can be shown on the basis of the linear tube theory that the oscillator is very nearly equivalent to the simple tuned circuit just considered. To bring this agreement about the resistance R must be determined by the identities

$$R \equiv \frac{Z_0}{1 - \alpha \dfrac{Z_0}{R_p}}$$
$$\alpha \equiv \frac{n_g}{n_p} - 1 \tag{4}$$

where Z_0 is the antiresonant impedance of the plate tuned circuit, R_p is the statically determined plate resistance $1/(\partial i_p/\partial e_p)$, and n_g/n_p is the grid-to-plate voltage ratio, and μ is the amplification factor.

Putting in the requirement that R be infinite when the amplitude has its steady state value a_0, and assuming that the mutual conductance of the vacuum tube varies linearly with $e_p + \mu e_g$, this reduces to

$$R = \frac{1}{\alpha\xi[f(t) - \beta(a - a_0)]} \tag{5}$$

where ξ is the fractional change in $1/R_p$ for a unit change in grid voltage, β is the ratio of the rectifier output to the plate swing, and $f(t)$ is the modulating grid signal.

Substituting this in (3), there results,

$$\frac{da}{dt} = \frac{-p}{2Q}\xi a[f(t) - \beta(a - a_0)]. \tag{6}$$

Making the substitution

$$\gamma \equiv a/a_0,$$

(γ is the fractional amplitude), this becomes,

$$\frac{d\gamma}{dt} = \frac{pa_0\xi}{2Q}\gamma\left[\beta(\gamma - 1) + \frac{f(t)}{a_0}\right] \tag{7}$$

a differential equation giving γ as a function of t. In particular, if

$$f(t) = \mu\beta a_0 \sin\theta \tag{8}$$

where $\theta \equiv qt$ and μ is the fractional modulation for infinitely low q, this becomes,

$$\frac{d\gamma}{d\theta} = \frac{1}{2Q}\,\frac{p}{q}\,\beta a_0 \xi \gamma [(\gamma - 1) + \mu \sin \theta]. \tag{9}$$

The boundary condition, that the function be periodic, leads to serious practical and perhaps theoretical difficulties when conventional power series methods are applied to the solution of this equation.

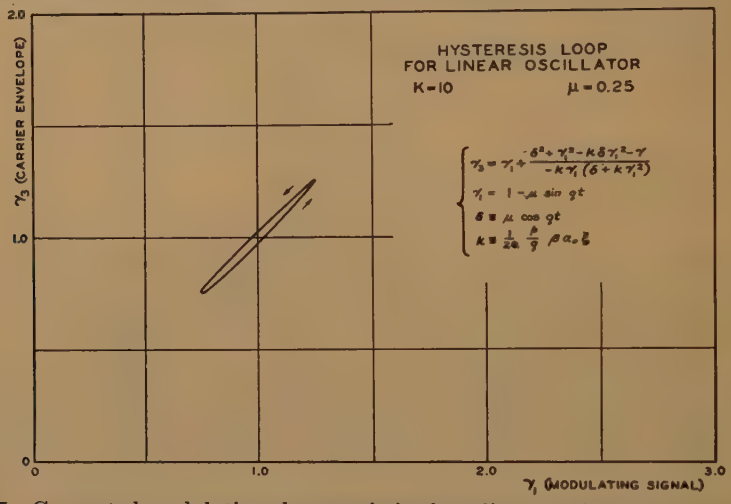


Fig. 7—Computed modulation characteristics for a linear oscillator. Modulation at 25 per cent.

Rewriting the equation in the form

$$\gamma = 1 - \mu \sin \theta + \frac{1}{k\gamma}\,\frac{d\gamma}{d\theta} \tag{10}$$

when,

$$k \equiv \frac{1}{2Q}\,\frac{p}{q}\,\beta a_0 \xi$$

suggests the possibility of a method of successive approximations. Setting

$$\frac{1}{k\gamma}\,\frac{d\gamma}{d\theta} = 0$$

there results,

$$\gamma_1 = 1 - \mu \sin \theta \tag{11}$$

the subscript indicating that the result is a first approximation. Substituting this result in the right-hand member of (10) there results,

$$\gamma_2 = \gamma_1 - \frac{\delta}{k\gamma_1} \tag{12}$$

where,

$$\gamma_1 \equiv 1 - \mu \sin \theta$$

$$\delta \equiv \mu \cos \theta.$$

Repeating the process,

$$\gamma_3 = \gamma_1 + \frac{k\delta\gamma_1^2 + \delta^2 + \gamma_1^2 - \gamma_1}{k\gamma_1(\delta - k\gamma_1^2)}. \tag{13}$$

Higher order approximations can be written down after a short amount of formal work.

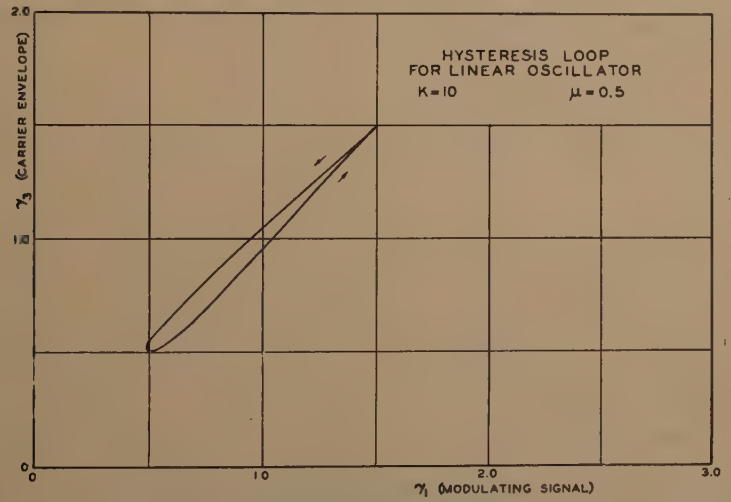


Fig. 8—Computed modulation characteristics for a linear oscillator. Modulation at 50 per cent.

Figs. 7 and 8 show typical curves corresponding to the expression, (13). They correspond well with curves determined experimentally. It will be noticed that the factor of merit in making dynamic effects minimum is the constant k, which decreases with the ratio of carrier to modulating frequency, p/q, with the damping of the tuned circuit, $1/Q$, and with the factor $\beta a_0 \xi$ which represents the fractional change in plate resistance due to a change in grid voltage equal to the rectifier output.

Conclusion

It has been seen that an oscillator, where the change in plate resistance is brought about by an automatically shifted bias, conforms closely to linear theory. Computations show that such an oscillator has high frequency stability and linear modulating characteristics. It has been

shown that dynamic effects can be minimized by a proper choice of the constant, k. In the practical design of an oscillator, a compromise is usually necessary between freedom from frequency modulation and freedom from dynamic hysteresis. A desirable solution to both of these troubles can usually be obtained by choosing a sufficiently high proportionality factor between the rectifier output and the amplitude of swing. This sharper control of equilibrium conditions is the essential advantage of the linear oscillator which has been described.

Acknowledgment

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MEASUREMENT OF THE FREQUENCY OF ULTRA-RADIO WAVES*

By

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Summary—*An analysis of a particular Lecher wire system whose characteristic impedance matches its input impedance and whose output end is short circuited, has led to an equation permitting the measurement of ultra-radio frequencies to three significant figures. The method is independent of end effects and has been applied to a determination of the velocity of propagation along iron wires.*

Introduction

FREQUENCY determinations in the short-wave radio range are based on direct measurements of the position of current or voltage nodes and loops along two parallel wires. As customarily carried out, the distance between successive nodes or loops along the "Lecher" wires is measured by means of a thermocouple galvanometer. This distance is equal to half of the wavelength, but the precise location of nodes and loops becomes increasingly difficult at the shorter wavelengths. In measurements on the permeability of iron for wavelengths below one meter[1] it was found that greater precision was required than could be obtained by the usual method. With a view to obtaining the requisite precision, an analysis of the particular Lecher wire system in use was undertaken, and has yielded a practical method which is accurate to three significant figures.

Let us consider, first, the simple cable theory which has been developed by many men and which is clearly presented in Page and Adams' "Principles of Electricity," pages 540 to 550. Fig. 1 shows a generator E, of internal impedance Z_0 which produces a pure sine wave voltage and sends current through the Lecher wires of length S which are terminated by the impedance Z_s. This current, whose value is i at a point s centimeters from the source, is the resultant of currents reflected back and forth along the wires between the two terminal impedances. The value of i varies sinusoidally with time and also varies in amplitude along the wires. The latter, amplitude value i_{s0} is given in (1). If the applied potential is a sine wave, the actual current i is obtained by multiplying equation (1) by ϵ^{jwt} and taking the imaginary part in the usual way.

* Decimal classification: R212. Original manuscript received by the Institute, September 2, 1932.

[1] J. Barton Hoag and Haydn Jones, *Phys. Rev.*, vol. 42, p. 571, (1932).

$$i_{s0} = \frac{E_0}{Z_k + Z_0}[\epsilon^{\gamma s} + \epsilon^{\gamma(2S-s)}T + \epsilon^{\gamma(2S+s)}DT + \epsilon^{\gamma(4S-s)}DT^2 + \epsilon^{\gamma(4S+s)}D^2T^2 + \cdots] \quad (1)$$

The first term in this equation represents the current which leaves the generator and travels toward the output end. The second term repre-

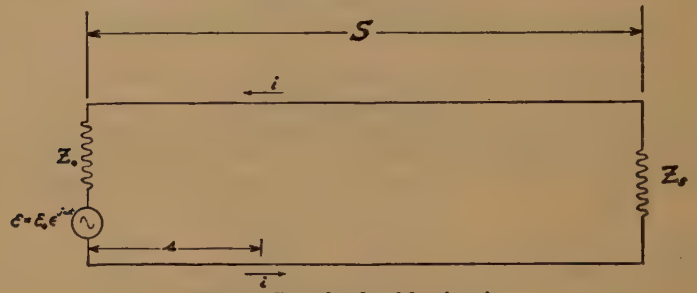

Fig. 1—Standard cable circuit.

sents the current which has been reflected and is returning to the source. The third represents that reflected from the input end, and so on. In the equation

E_0 = the maximum voltage of the generator

Z_0 = the impedance of the generator

Z_k = the characteristic line impedance

$= \sqrt{Z_L Z_C}$,

where,

Z_L = the series impedance of one centimeter of the double wires

$= R + j\omega L$,

Z_C = the parellel impedance of one centimeter of the double wires

$= 1/(G + j\omega C)$.

R, L, G, and C are the resistance, inductance, parallel conductance, and capacitance, respectively, per centimeter of the double wires.

$\omega = 2\pi f$

f = the frequency of the source in cycles per second.

$j = \sqrt{-1}$

$D = (Z_k - Z_0)/(Z_k + Z_0)$

$T = (Z_k - Z_s)/(Z_k + Z_s)$

γ = the propagation constant.

$$= -\sqrt{Z_L/Z_C}$$

$$= -\alpha - j\phi$$

α = the attenuation constant per centimeter of the Lecher wires

$$= \sqrt{[(RG - \omega^2 LC) + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}]/2}$$

ϕ = the phase constant per centimeter of the Lecher wires

$$= \sqrt{[-(RG - \omega^2 LC) + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}]/2}$$

$$= 2\pi/\lambda = 2\pi f/v = \omega/v$$

λ = the wavelength in centimeters in the Lecher wires.

v = the velocity of propagation of the current along the wires.

For nonmagnetic wires, the wavelength λ is practically the same as that of the radiated electromagnetic waves, and the velocity v is nearly the same as the velocity of light (3×10^{10} centimeters per second). For iron wires, both λ and v are appreciably smaller.

A Special Case—Matched Input and Short-Circuited Output

It is possible, experimentally, to make the input impedance Z_0 equal to the line impedance Z_k. If this is done $D=0$, and all terms in (1) drop out except the first two. This means that the current travels to the output end and is reflected back to the source where it is absorbed. Equation (1) thus reduces to

$$i_{s0} = \frac{E_0}{2Z_k}[e^{\gamma s} + e^{\gamma(2S-s)}T]. \qquad (2)$$

In Fig. 2, a magnetron oscillator is used to produce the ultra-radio frequencies. The system inside the dotted line may be replaced to a first approximation by a sinusoidal oscillator in series with an input impedance Z_0. This impedance consists essentially of the condensers c and the capacity between the filament f and anode a of the magnetron. For the apparatus used this impedance was approximately equal to the impedance Z_k of the parallel Lecher wires WW.

The short-circuiting bridge, as described by Tonks[2] and shown at B in Fig. 2, serves as a practically complete short circuit at the output and thus eliminates the effect of currents induced in the wires to the right of B and gives complete reflection of the waves. It then follows that $Z_s=0$, and $T=1$, so that

[2] L. Tonks, "Impedance characteristics of loaded Lecher systems," *Physics*, vol. 2, p. 1; January, (1932).

$$i_{s0} = \frac{E_0}{2Z_k}\left[\epsilon^{\gamma s} + \epsilon^{\gamma(2S-s)}\right]. \tag{3}$$

It is obvious that the leakage conductance G may be made negligible. If we assume that $R \ll \omega L$, the attenuation α is practically zero, so that we may take $\gamma = -j\phi$.

Then,

$$i_{s0} = \frac{E_0}{2Z_k}\left[\epsilon^{-i\phi s} + \epsilon^{-i\phi(2S-s)}\right]. \tag{4}$$

Multiplying by $\epsilon^{i\omega t}$ and taking the imaginary part, gives the actual current,

$$i = K_1 \sin(\omega t - \phi s) + \sin(\omega t - \phi l) \tag{5}$$

where K_1 is a constant and $l = S - s$.

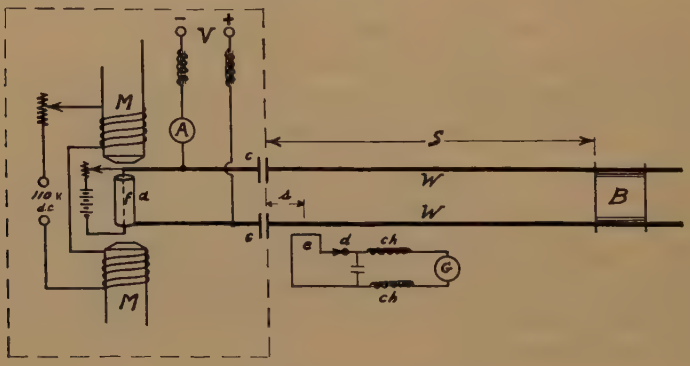

Fig. 2—Experimental circuit.

As a result of this high-frequency alternating current, an e.m.f. of amount e will be induced in the short pick-up wire located near the Lecher wires at s. Since e is proportional to the time rate of change of the current, we have

$$e = K_2(\cos \phi s + \cos \phi l) \cos \omega t + (\sin \phi s + \sin \phi l) \sin \omega t \tag{6}$$

where K_2 is a constant. This voltage drives a current through a rectifier such as the crystal d of Fig. 2. This current may be expressed as a power series in the following fashion

$$i_g' = ae + be^2 + ce^3 + \cdots \tag{7}$$

in which a, b, c, $\cdots$ are constants. The direct-current component of this current now actuates the galvanometer G while the alternating-current components are by-passed through a small condenser. Substituting (6) in (7), it is seen that the terms with odd powers of e yield

alternating currents. However, since $\cos^2\omega t = (1+\cos 2\omega t)/2$ and $\sin^2\omega t = (1-\cos 2\omega t)/2$, the second term, be^2, gives the useful direct current

$$i_g = \frac{bK_2^2}{2}(\cos \phi s + \cos \phi l)^2 + (\sin \phi s + \sin \phi l)^2. \quad (8)$$

Since $\sin^2\phi s + \cos^2 \phi s = 1$, and $l = S - s$, the equation reduces to

$$i_g = K \cos^2 \phi(S - s) \quad (9)$$

where K is a constant. As previously stated, $\phi = 2\pi/\lambda$ in radian measure. In degrees, $\phi = 360/\lambda$. Therefore, we arrive at a simple equation

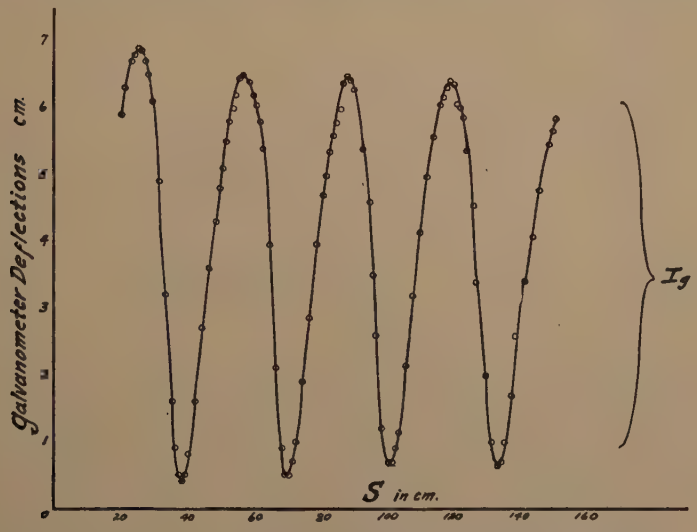

Fig. 3—Experimental data. Wavelength about 64 centimeters.

between the galvanometer deflections (which are proportional to i_g) and the position S of Tonk's bridge.

$$i_g = K \cos^2 \frac{360(S - s)}{\lambda}. \quad (10)$$

Maximum values occur when $360(S-s)/\lambda = 0°$, 180°, 360°, etc., the distances between successive maxima being equal to $\lambda/2$, as expected. Since the greatest value of the cosine squared is 1, it is seen that K represents the maximum current in the galvanometer, I_g. Then

$$i_g = I_g \cos^2 \frac{360(S - s)}{\lambda}. \quad (11)$$

Solving this equation for S, we have

$$S = s + \frac{\lambda}{360} \cos^{-1} \sqrt{\frac{i_g}{I_g}}. \tag{12}$$

This equation may be compared to that of a straight line

$$y = b + mx \tag{13}$$

where y corresponds to S, b to s (the y intercept), m to $\lambda/360$ (the slope), and x to $\cos^{-1}\sqrt{i_g/I_g}$. If then, we plot values of $\cos^{-1}\sqrt{i_g/I_g}$ as abscissas and values of S as ordinates, a straight line should result whose

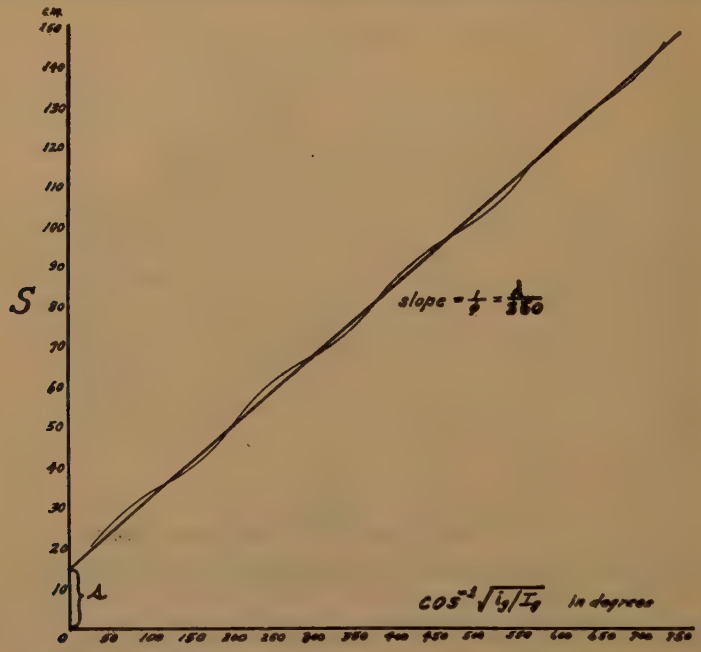

Fig. 4—Replot of Fig. 3. Wavelength = 63.81 centimeters.

y intercept would be s and whose slope would be $\lambda/360$. Therefore, by plotting in this fashion and measuring the slope of the straight line, a value of λ may be accurately determined which is uninfluenced by end effects, bridge short circuit, etc., which are, in a practical case to be included in s.

Results

To illustrate this method, an experimental set of values of galvanometer deflections are plotted against bridge positions in Fig. 3. These data, replotted in the manner indicated above, are shown in Fig. 4. as

the wavy line whose average is a straight line, as predicted. In plotting Fig. 4, each value of i_g was divided by that value of I_g corresponding to the nearest peak. As will be shown below, a more suitable value of I_g may be deduced from the limiting lines shown in Fig. 3.

It is important that the frequency and strength of the source remain constant during the time needed to obtain a complete set of data. This time may be shortened without undue sacrifice of accuracy by limiting the data to two peaks at opposite ends of the Lecher wires. Rougher values may also be obtained from only a portion of one peak, a distinct advantage over the usual method.

Wavelengths calculated from a complete set of data, as in Figs. 3 and 4, are found to be reliable to three significant figures. The accuracy is fully ten times that of the usual peak-to-peak method for this ultra-radio region.

The source was next maintained at a constant frequency and intensity while wavelength measurements were made, first with brass Lecher wires and then with iron wires. The following are illustrative values of the wavelengths, in centimeters.

Brass:	$\lambda_B = 63.81$,	50.83,	36.13,	22.21
Iron:	$\lambda_F = 63.55$,	50.68,	36.04,	22.18

Assuming that the velocity of propagation in the brass is essentially the velocity of light (2.998×10^{10} centimeters per second), the velocities in iron, computed from $f\lambda_B = 2.998 \times 10^{10}$ and $f\lambda_F = v$ are 2.986, 2.989, 2.990, and 2.994×10^{10} centimeters per second, respectively, for the decreasing brass wavelengths above. The frequency of the source is the same regardless of the metal used in the Lecher wires and is identical with that of the radiated energy. From the equations just given, we have the respective frequency values 469.8, 589.8, 829.8, and 1350. megacycles.

Other Operating Conditions

In the method for wavelength or frequency measurement presented above, it has been assumed that the input impedance is exactly equal to the characteristic impedance Z_k of the Lecher wires. In order to accomplish this, the source, whether of the magnetron type or any other, might be inductively coupled to the Lecher wires by means of a suitable coil and condenser system. An approximate calculation for Z_k may be made from the equations given in the Introduction. If the input is not matched with the line impedance but all other conditions previously described are maintained, it will be found by an analysis similar to the one presented above, that

$$i_g = [K_3 \cos^2 \phi(S - s)][1 + 2D \cos 2\phi S] \tag{14}$$

where K_3 is a constant and $D=(Z_k-Z_0)/(Z_k+Z_0)$. This may be expressed as

$$i_g = I_g \cos^2 \phi(S - s) \tag{15}$$

where the maximum value, $I_g=K_3(1+2D \cos 2\phi s)$ is seen to vary sinusoidally. This probably accounts for the fact that the experimental points in Fig. 4 gave a wavy, instead of a perfectly straight line.

It was assumed that the attenuation was zero. If this is not true, the factor α must be included in the equations. If we assume that we have a matched input, short-circuited output, and negligible transconductance as before, and that terms in α squared or higher are very small, we arrive at the equation

$$\begin{aligned} i_g &= [K_4 \cos^2 \phi(S - s)][1 - 2\alpha S] \\ &= I_g \cos^2 \phi(S - s) \end{aligned} \tag{16}$$

where K_4 is a constant. The maximum current I_g is here seen to decrease as the distance S is increased. This appears in Fig. 3. It would, therefore, seem more satisfactory that, for a given position S, we choose the corresponding value of I_g from the limiting lines instead of from the nearest peak. For the case under discussion, terms in $(R/\omega L)^4$ may be dropped, so that the expressions for α and ϕ in the Introduction become

$$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}} \quad \text{and} \quad \phi = \sqrt{\omega^2 LC + \alpha^2}. \tag{17}$$

The method adopted by L. Sokolow[3] and by R. Michels[4] was to match the output rather than the input end of the Lecher wires. In this case, (1) reduces to

$$i_g = \frac{E_0}{Z_k + Z_0} \epsilon^{-2\alpha s} \tag{18}$$

where $\alpha=R/2\sqrt{C/L}$, and the position of the pick-up loop is the variable.

[3] L. Sokolow, *Ann der Phys.* vol. 83, p. 1136, (1927).
[4] R. Michels, *Ann der Phys.* vol. 8, p. 877, (1931).

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FREQUENCY DOUBLING IN A TRIODE VACUUM TUBE CIRCUIT*

By

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Summary—*This paper gives a quantitative analysis of operating performance of a triode vacuum tube as a frequency doubler. With slight changes this analysis can be applied to tripling, quadrupling, etc. Three methods of attack have been outlined: theoretical analysis, graphical solution, and experimental results. The theoretical analysis furnishes a general solution to the problem, from which general conclusions as to the best operating conditions can be drawn. The graphical solution furnishes a simple method by which rapid calculations can be made for a particular case. The experimental results are valuable for comparison because they give the actual results of operation.*

The primary object of this work has been to investigate the conditions that will give maximum plate efficiency and consequently the most desirable operating conditions, also keeping in mind that power output and power amplification are important factors to consider in the practical application of the frequency doubler.

ALTHOUGH there are numerous publications on the subject of frequency multiplication there are only a very few that treat the subject quantitatively. Perhaps the most thorough theoretical treatment of this subject is that by J. Marique,[1] which is essentially a graphical analysis. W. Bunimowitsch[2] has published a very good article of a practical nature on this subject, and R. Mesny[3] has published a short theoretical analysis which is similar in its method of attack to the analysis developed in this paper.

Only a simple circuit with an antiresonant tank circuit in the plate lead tuned to the second harmonic is treated in this article. No doubt certain changes in the circuit would be advantageous, such as introducing an antiresonant circuit, of small physical dimensions, in the grid lead, and coupled to the tank circuit so that the second harmonic will be fed into the grid circuit in addition to the fundamental which must always be in such proportion that it will control the frequency in the tank circuit of the doubler. Another scheme that appears favorable is to use a double-grid vacuum tube that has symmetrical grids, with re-

* Decimal classification: R357. Original manuscript received by the Institute, July 25, 1932.

[1] Jean Marique "Note sur le calcul des etages multiplicateurs de frequence a triodes," *L'Onde Electrique*. vol. 8, p. 1; January, (1929).

[2] Wladimir Bunimowitsch, "Ueber Frequensverdopplung mitElektronenrehren," Zeit. für Hochfrequenz, vol. 35, p. 223; June, (1930).

[3] Rene Mesny, "Au subject de la multiplication des frequences par les triodes," *L'Onde Electrique*, vol. 9, p. 18; January, (1930).

spect to the plate and filament, intermeshed in the same plane. With these grids connected to the ends of a pick-up coil which is by-passed from its center to the filament and having an antiresonant circuit in the plate lead tuned to the second harmonic, the fundamental will be balanced out completely in the plate circuit. This should give a condition of high plate efficiency.

Theory of Doubler Operation

The frequency doubler operates in many respects like a class C amplifier. The class C amplifier operates with a grid-bias voltage greater than the voltage necessary to block the plate current. When biased this way it can be used as an efficient radio-frequency amplifier for continuous waves, as well as for a modulator. It has a higher plate efficiency (direct to alternating-current conversion) than the class A amplifier which operates on the linear portion of the grid-voltage, plate-current characteristic. It also gives a higher power output, with lower gain and has considerable harmonic distortion present.

The essential difference between a frequency doubler and a class C amplifier is that the tank circuit is tuned to the second harmonic instead of the fundamental. When tuned to the second harmonic, the tank circuit presents a high impedance load at this harmonic but not at the fundamental or other harmonics. Consequently, practically all the power output will be at the second harmonic.

One of the main uses of the frequency doubler, as well as the class C amplifier, is for transmitting purposes. Besides increasing the power, the doubler multiplies the frequency and eliminates the necessity of neutralizing back coupling which is ordinarily present in the radio-frequency amplifiers. Since piezo-electric quartz crystals, which are usually used to control the frequency of transmitters, become fragile when ground for very high frequencies, it is a better policy to select a crystal that will oscillate at a lower frequency and then, by means of frequency multiplication in vacuum tube circuits, multiply the frequency to that at which the set is to radiate.

An advantage when multiplying the frequency is, that under ordinary circumstances, the circuit will not oscillate due to back coupling, for the plate and grid circuits are tuned to different frequencies. However, feed-back to the grid circuit through the elements of the vacuum tube may cause self-excitation if the bias voltage is produced by a low voltage grid-bias battery or produced by a high grid-leak resistance. With high bias voltages produced by batteries and operating the vacuum tube so that all the sloping portion of the tube characteristic is utilized, no self-oscillation should be experienced. Feed-back is an ad-

vantage if it is small and coincides with the natural frequency of the grid circuit. As mentioned above this condition can be realized if an antiresonant circuit of small physical dimensions is placed in the grid lead and tuned to the second harmonic. In any case the feed-back can be eliminated by neutralization if it becomes objectionable by causing self-oscillation.

When frequency doublers or class C amplifiers are used for transmitting purposes, it is desirable to have the maximum power delivered from each tube. Perhaps the most important factor is the plate efficiency, for usually the power output is limited by the heating of the plate. In addition to this, the cost of direct-current power for the plate supply becomes quite appreciable for large transmitters, so high plate efficiency is also an important factor from an economic view point.

With high plate efficiency as the criterion for good operating conditions, it will be worth while to investigate more thoroughly the conditions that will bring it about. The power dissipated at any instant in a

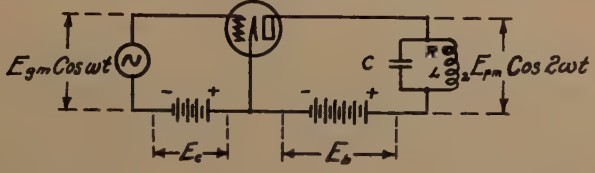

Fig. 1—Basic circuit.

vacuum tube is primarily due to the product of the instantaneous plate current by the instantaneous plate voltage. It has been found that for high plate efficiency, it is desirable to adjust the operating voltages of the vacuum tube so that the plate current will flow for only part of a cycle. This plate current flowing through the external load impedance sets up a voltage across this impedance at the harmonic frequency to which the tank circuit is tuned, which causes the instantaneous plate voltage to drop to a minimum when the plate current is a maximum. Consequently, the average product of the instantaneous plate voltage by the instantaneous plate current over a cycle is a minimum and the tank circuit is kept in continuous oscillation even though the energy is being supplied in pulses. These conditions are very important for high plate efficiency.

Theoretical Analysis

Fig. 1 shows the basic circuit used in this analysis. Fig. 2 shows a plot of the grid-voltage, plate-current static characteristics, with the assumption that they are straight lines. It also shows the instantaneous values of the grid and plate voltages, with the assumption that they

are sinusoidal and the resultant instantaneous plate current for this basic circuit. In this particular case the grid voltage never becomes positive with respect to the filament so the grid current is zero over the whole cycle.

The assumption that the static characteristics are straight lines is due to the fact that it is very difficult to write an exact expression for the plate current in terms of the grid and plate voltages. Without this assumption it is very difficult to draw any general conclusions as to the effect of the various parameters on the plate efficiency and power output. Since the alternating-current grid voltage is usually large enough to swing over most of the straight portion of the vacuum tube characteristic, there is very little error introduced by neglecting the curvature near the cut-off point. However, when the maximum value of the instantaneous grid voltage approaches the minimum value of the instantaneous plate voltage this assumption is no longer approximately true, because the mutual conductance of the vacuum tube becomes a variable for this condition of operation.

The alternating-current grid voltage will actually be sinusoidal if the output voltage of the driving generator is sinusoidal. If the resistance in the tank circuit is very small (high-Q circuit) the alternating-current plate voltage, which depends largely on the second harmonic current flowing in this circuit, will be essentially sinusoidal. However, if a high "Q" circuit is not used in the plate lead the damping effect will be appreciable. Thus, the alternate cycles which do not receive energy will have a decreased amplitude.

In this analysis grid current loss is not considered. In any practical case it is desirable to keep the grid current as low as possible, and still maintain good operating conditions, but for highest plate efficiency it is necessary to draw appreciable grid current which will help to limit the maximum power output from the tube.

The filament power is neglected because it ordinarily is not a parameter to be varied and the power in this form is not as expensive as the direct-current plate power.

Referring to Fig. 2 it will be noted that the instantaneous grid voltage, e_g, is the algebraic sum of the grid-bias voltage and the alternating-current grid voltage, which may be written as follows:

$$e_g = -E_c + E_{gm} \cos (wt). \qquad (1)$$

Referring again to Fig. 2 it will be noted that the instantaneous plate voltage, e_p, is the algebraic sum of the direct-current plate battery voltage and the alternating-current plate voltage at the double frequency which may be written as follows:

$$e_p = E_b - {}_2E_{pm} \cos (2wt). \tag{2}$$

Since the pulses of plate current, as shown in Fig. 2 are a recurrent phenonemon the instantaneous plate current can be expressed in the form of Fourier's series. By making the current symmetrical about the ordinate axis, as shown in Fig. 2, the sine terms will be eliminated and the equation of the instantaneous plate current becomes:

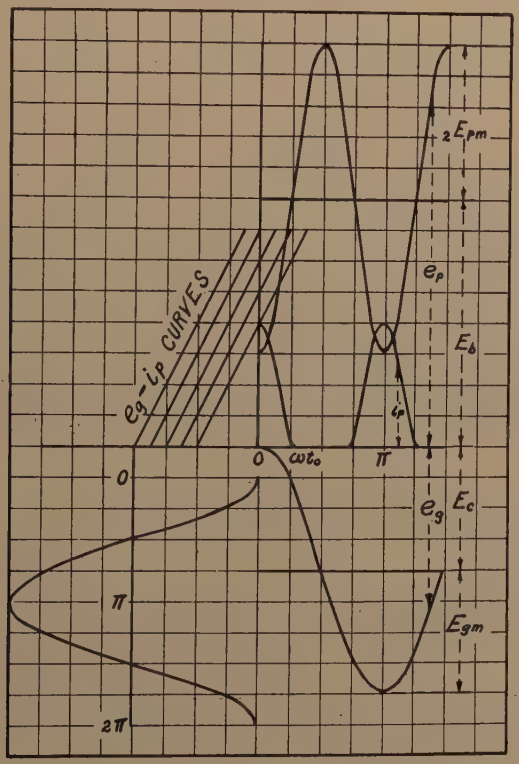

Fig. 2—Graphical representation of equations (1), (2), and (6).

$$i_p = {}_0I_p + {}_1I_{pm} \cos (wt) + {}_2I_{pm} \cos (2wt) + \cdots + {}_nI_{pm} \cos (nwt) + \cdots . \tag{3}$$

Where,

$${}_0I_p = 1/\pi \int_0^{wt_0} i_p \, d\,(wt) \tag{4}$$

$${}_nI_{pm} = 2/\pi \int_0^{wt_0} i_p \cos (nwt) \, d\,(wt). \tag{5}$$

The equation of the instantaneous plate current becomes:

$$i_p = g_m \left\{ e_g + \frac{e_p}{u} \right\}.$$

The above equation may be written:

$$i_p = \frac{1}{r_p} \left\{ E_b - uE_c + uE_{gm} \cos(wt) - {}_2I_{pm}\, r \cos(2wt) \right\}. \qquad (6)$$

Where,

r_p is the plate resistance of the tube when operated as a class A amplifier.

$r = L/r_2C$ is the load resistance expressed by the well-known formula.

Equation (6) is true for positive values of the right-hand side up to the point of saturation, that is, over the straight portion of the static characteristic, and when the right-hand side becomes negative the instantaneous plate current i_p should be written equal to zero.

The instantaneous plate current becomes zero at the time $t = t_0$, that is, $wt = wt_0$ as shown in Fig. 2. For this condition, (6) becomes:

$${}_2I_{pm} = \frac{E_b - uE_c + uE_{gm} \cos(wt_0)}{r \cos(2wt_0)}. \qquad (7)$$

Since this is the maximum value of the second harmonic current in the plate circuit the output power takes the form:

$$P_0 = \frac{r}{2}\, {}_2I_{pm}{}^2 = \frac{r}{2} \left\{ \frac{E_b - uE_c + uE_{gm} \cos(wt_0)}{r \cos(2wt_0)} \right\}^2. \qquad (8)$$

Substituting the instantaneous plate current as given in (6), in (4) one can obtain an expression for the average direct current in the plate circuit, which is:

$$I_p = \frac{1}{\pi r_p} \left\{ E_b\,(wt_0) - uE_c\,(wt_0) + uE_{gm} \sin(wt_0) - \frac{{}_2I_{pm}\, r}{2} \sin\,(2wt_0) \right\}. \qquad (9)$$

The power input to the plate circuit may now be expressed by the following equation:

$$P_I = {}_0I_p\, E_b = \frac{E_b}{\pi r_p}\{E_b\,(wt_0) - uE_c\,(wt_0) + uE_{gm}\sin(wt_0)$$
$$-1/2E_b\tan(2wt_0) + 1/2uE_c\tan(2wt_0)$$
$$-1/2uE_{gm}\cos(wt_0)\tan(2wt_0)\}. \qquad (10)$$

Equations (8) and (10) may now be used to express the efficiency of the plate circuit in per cent. Thus:

$$\text{Eff.}_p = \frac{P_0}{P_I}100 = \frac{r_p N^2}{2E_b r\mathcal{D}D}100. \qquad (11)$$

Where,

$$N = E_b - uE_c + uE_{gm}\cos(wt_0) \qquad (12)$$

$$\mathcal{D} = \cos(2wt_0) \qquad (13)$$

$$D = E_b\,(wt_0)\cos(2wt_0) - uE_c\,(wt_0)\cos(2wt_0)$$
$$+ uE_{gm}\sin(wt_0)\cos(2wt_0) - 1/2E_b\sin(2wt_0)$$
$$+ 1/2uE_c\sin(2wt_0) - 1/2uE_{gm}\cos(wt_0)\sin(2wt_0). \qquad (14)$$

In order to make a study of the influence of the independent parameters such as the effective resistance of the tuned tank circuit and the plate resistance of the tube it is possible to eliminate the dependent parameter ${}_2I_{pm}$.

Substituting the instantaneous value of the plate current as given in (6) into (5) and then substitute for ${}_2I_{pm}$ as given in (7) results in the following equation:

$$m\pi + uE_{gm}\pi\cos(wt_0) + \frac{r}{r_p}\{Am + uE_{gm}B\} = 0. \qquad (15)$$

Where,

$$m = E_b - uE_c$$
$$A = (wt_0) - 1/4\sin(4wt_0) \qquad (16)$$

$$B = (wt_0)\cos(wt_0) + 1/3\sin(wt_0)$$
$$-3/8\sin(3wt_0) - 1/24\sin(5wt_0). \qquad (17)$$

The curves of A and B as functions of (wt_0) have been worked out and plotted in Fig. 3.

Solving (15) for E_{gm} the following expression is obtained:

$$E_{gm} = -\frac{m\pi + m(r/r_p)\,A}{u\pi\cos(wt_0) + (r/r_p)\,uB}. \qquad (18)$$

Fig. 4 shows the value of E_{gm} plotted as a function of (wt_0) for a

particular value of m and various ratios of r/r_p. It will be noted that all the curves intersect at the point $wt_0 = 45$ degrees.

Now, solving (15) for the plate load resistance gives:

$$r = -\frac{\pi r_p[m + uE_{gm}\cos(wt_0)]}{Am + uE_{gm}B}. \tag{19}$$

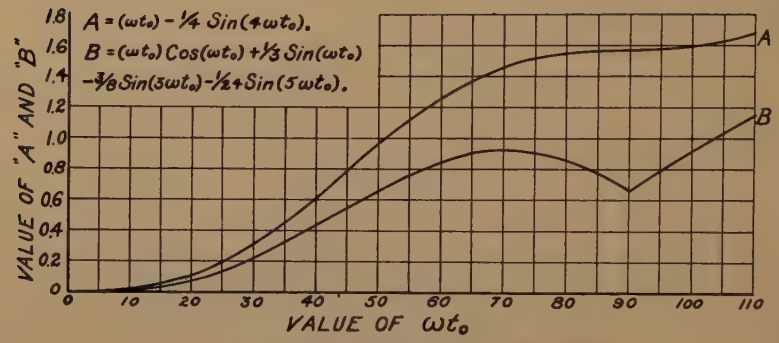

Fig. 3—Solution of equations (16) and (17).

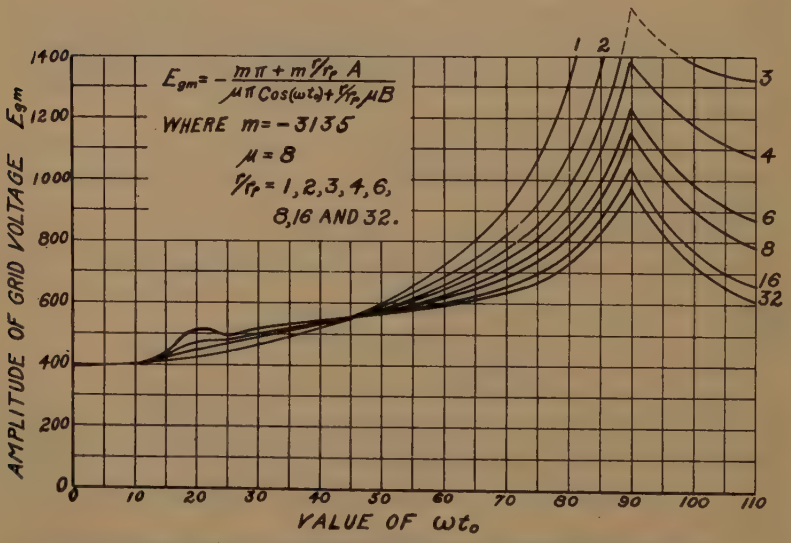

Fig. 4—Solution of equation (18) for a particular value of m and various ratios of r/r_p.

Substituting this in (11) one secures,

$$\text{Eff.}_p = \frac{-100\ \mathcal{N}N}{2E_b\ \mathcal{D}D}, \tag{20}$$

where,

$$\mathcal{N} = Am + uE_{gm}B. \tag{21}$$

In (20) the per cent plate efficiency is expressed in terms of u, E_b, E_c, E_{gm}, and (wt_0). For a particular tube u is constant so we may write:

$$\frac{\partial}{\partial E_b} (\text{Eff.}_p) = 0 \tag{22}$$

$$\frac{\partial}{\partial E_c} (\text{Eff.}_p) = 0 \tag{23}$$

$$\frac{\partial}{\partial E_{gm}} (\text{Eff.}_p) = 0 \tag{24}$$

$$\frac{\partial}{\partial (wt_0)} (\text{Eff.}_p) = 0. \tag{25}$$

The solution of these four equations should give the condition of maximum plate efficiency. Since the solution of these equations is rather tedious the work will be omitted here but for practical purposes the following results are valuable.

$$(wt_0) = 45 \text{ degrees} \tag{26}$$

$$E_{gm} = \frac{-m\sqrt{2}}{u}$$

or,

$$E_b = \frac{2uE_c - \sqrt{2}\,uE_{gm}}{2}. \tag{27}$$

Equation (26) says that for maximum plate efficiency the angle of cut-off of plate current must be 45 degrees, and (27) gives the relation between E_b, E_c, and E_{gm}.

For the important condition that $E_c = E_{gm}$, that is, the condition for maximum power output without drawing grid current, (27) becomes:

$$E_b = uE_c\left(\frac{2-\sqrt{2}}{2}\right). \tag{28}$$

Graphical Solution

It has already been pointed out that the graphical solution is a valuable tool in determining the best operating conditions for a particular case. Many workers get excellent tube performance by merely arranging the material at hand into a circuit and making adjustments by a little experimenting. This works very well for small circuits; but is an expensive procedure for high powered sets.

Practical problems usually arise in such a way that definite voltages, currents, and power are required; but it will be found impossible to tell directly what should be done to produce these results in a doubler. One way to answer this practical question is to make a graphical solution and from this solution choose the most desirable operating conditions for the vacuum tube. Since there are comparatively few types of tubes in practice, one good set of calculations will cover a multitude of applications.

The method used here is an adaptation of the graphical method employed by D. C. Prince and F. B. Vodges.[4]

The first requirement for this method of attack is to obtain a family of static characteristic curves. By means of these curves one can de-

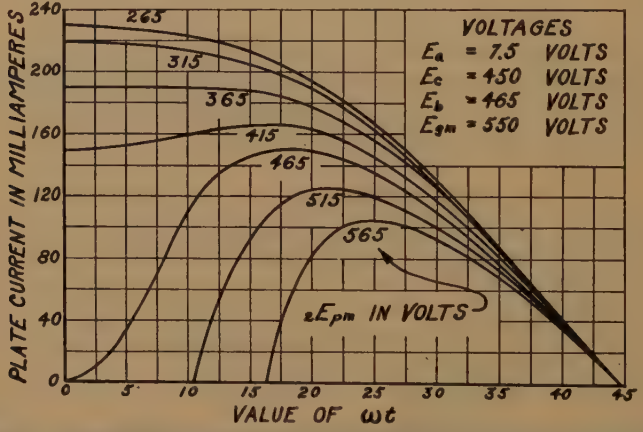

Fig. 5—Graphical solution to show instantaneous plate current vs. value of (wt) for a UX-210 as a doubler.

termine the instantaneous plate voltage, plate current, and grid current for various values of grid voltage and tank load combination. The output and plate efficiency under given conditions can be computed by this point-to-point method of analysis. By assuming several sets of operating conditions it is possible to make a selection which will give the most favorable results.

Applying these ideas more specifically, the assumption that the grid and plate voltages are sinusoidal permits the use of (1) and (2). Since the voltages are symmetrical about the ordinate axis, it is only necessary to make calculations for positive values of (wt_0). To facilitate the work it is best to make a table taking increments such as 5 or 10 degrees up to the angle (wt_0).

[4] D. C. Prince and F. B. Vodges, "Vacuum tubes as oscillation generators," *Gen. Elec. Rev.*, (1927)–(1928).

It is now desirable to obtain the power output, power input, plate efficiency, and resistance of the antiresonant tank circuit from the values in the table. First, it is necessary to compute the direct-current component and second-harmonic component. This is readily accomplished by means of the trapezoidal rule for evaluating a definite integral. With these values the other desired quantities can easily be computed.

For a particular case the curves of Fig. 5 were plotted and the resulting plate efficiency was plotted in Fig. 7 for purposes of comparison. It will be noted that for maximum plate efficiency the plate voltage ${}_2E_{pm}$ is equal to E_b. This means that at the time $t=0$ the instantaneous plate current is zero. This relationship is shown in Fig. 5.

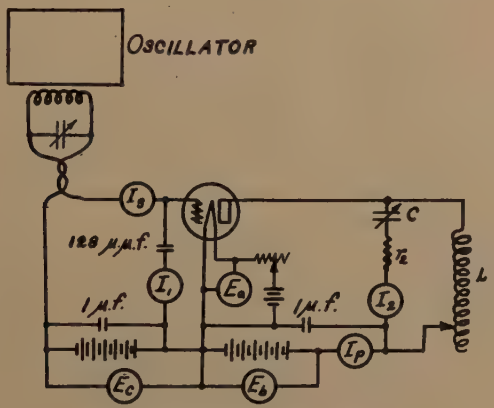

Fig. 6—Measuring arrangement.

While working at audio frequencies with an oscillograph connected in the plate circuit it was noted that for good plate efficiency there was a decided dip in the center of the plate current pulse. Evidently this condition holds true at radio frequencies.

Experimental Results

Experimental tests were made with a UX-210 vacuum tube. The measurements were made with a wavelength of $\lambda=406$ meters ($f=740$ kc) in the grid circuit, and $\lambda_2=203$ meters ($f_2=1480$ kc) in the plate circuit which had an antiresonant tank circuit tuned to the double frequency. The other parameters held constant were: amplification factor $=8$, direct-current plate voltage $E_b=465$ volts, direct-current grid-bias voltage $E_c=450$ volts, and the total resistance in the antiresonant tank circuit of $r_2=25.3$ ohms.

The measuring arrangement is shown in Fig. 6. The amplitude of

the grid voltage E_{gm} was determined by means of a thermal milliampere meter connected in series with a small known capacity. An ohm spun resistor of 25 ohms, which is practically noninductive, was placed in the antiresonant tank circuit in order that slight changes in the resistance of the tank circuit due to variation of the contact on the inductance would not appreciably change the "Q" of the circuit.

Holding E_{gm} constant at 540 volts the plate load was varied over a wide range of values. Similar sets of readings were taken for $E_{gm} = 550$

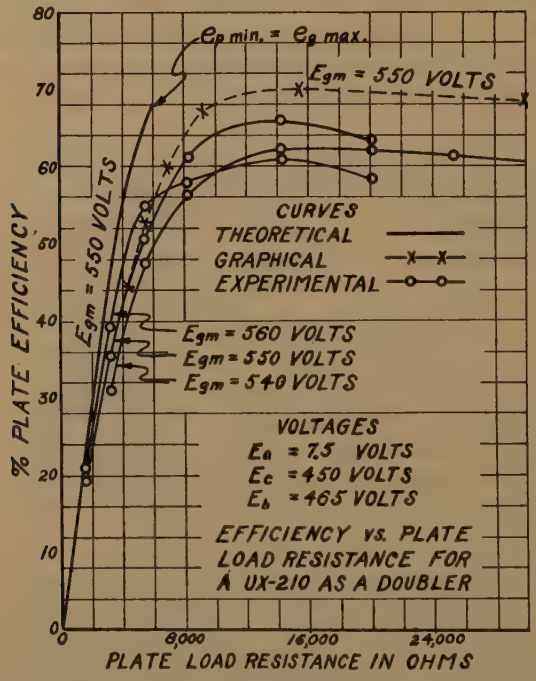

Fig. 7—Comparison of the results of the three methods of attack.

and 560 volts. The results of these three runs have been plotted in Fig. 7 for purposes of comparison with the results of the other methods of attack.

Comparison of the Three Methods of Attack

If one is interested primarily in plate efficiency then there is an amplitude of grid voltage that will give maximum efficiency. The curves in Fig. 7 show that higher efficiency is secured with $E_{gm} = 550$ volts than either of the other voltages. If $E_{gm} = 554$ volts the angle of cut-off of the plate current is $(wt_0) = 45$ degrees and the condition of maximum plate efficiency results.

The theoretical, graphical, and experimental results, as shown in

Fig. 7, agree very closely for low values of plate load resistance. Since the graphical solution takes account of the actual operating conditions, the results obtained agree quite closely with the experimental results over the whole range of values of the plate load resistance. The experimental curve for $E_{gm}=550$ volts agrees more nearly with the graphical results than either of the other two values of E_{gm}. The theoretical curve was only plotted up to the point where the minimum instantaneous plate voltage is equal to the maximum instantaneous grid voltage, for beyond this point the assumptions on which the analysis was based are no longer approximately true. The experimental and graphical curves start to bend over very rapidly when the condition that the minimum instantaneous plate voltage equals the maximum instantaneous grid voltage is approached.

General Conclusions

There are certain criteria that are essential if the doubler is to operate properly. Since the inherent characteristics of vacuum tubes are quite elaborate in their nature and play such an important rôle in the operation of efficient doubling circuits, it is necessary to understand their influence on the circuits in which they are to operate. For practical problems it is important to know what conditions will bring about the desired results.

It has already been pointed out that high plate efficiency is perhaps the most important factor to consider in the proper operation of the doubler. Along with high plate efficiency the power output will be a maximum for a given tube.

A very important special case is when a quartz crystal is used to control the frequency of a short-wave transmitter. The power delivered by the quartz crystal oscillator is very weak so it is essential that the grid be kept negative at all times. With this condition no power will be absorbed from the crystal oscillator circuit by the doubler. With this restriction on the doubler the best condition of operation is obtained when the maximum value of the grid voltage at the fundamental frequency is equal to the grid bias voltage. For this condition of operation the theoretical analysis will hold if the power dissipated in the plate is not too large. Equations (26) and (28) give the conditions for maximum plate efficiency. These parameters should be chosen so all the straight portion of the characteristic is utilized, provided, the power dissipated in the plate is not too large. This requirement is fulfilled at a sacrifice to the plate efficiency. Considerably higher plate efficiency would be realized if the grid were allowed to go positive.

The value of the plate load resistance is another very important factor in determining the best operating conditions. The minimum in-

stantaneous plate voltage and the maximum instantaneous grid voltage occur simultaneously in the middle of the plate current pulse as is illustrated in Fig. 2. Since the instantaneous plate efficiency is fixed by the plate voltage drop it is very important that the input should not be allowed to decrease for this should be the time of maximum efficiency and is therefore of the greatest importance. As the maximum plate current is obtained when the grid voltage is from 80 to 100 per cent of the plate voltage it is desirable to select a load resistance that will bring this condition about. Direct methods to calculate this value are quite tedious. The graphical solution offers an indirect method that is quite simple.

The maximum power output, which coincides with the maximum power amplification from the fundamental frequency input to the double frequency output, occurs when the load resistance in the plate circuit is approximately equal to the plate resistance of the tube. This is considerably lower than is necessary to give maximum plate efficiency.

There are certain important points in selecting a vacuum tube for a doubler. Since the power output for a given size tube is limited by heating, it is an advantage to use tubes with thoriated filaments. It is also desirable to use tubes that have a large mutual conductance and a low internal plate resistance.

Acknowledgment

In conclusion it is the writer's pleasant duty to thank Professor W. L. Everitt of the Electrical Engineering Department, the Ohio State University, for his encouragement and criticism of this work.

List of Symbols

E_a Filament supply voltage (direct current).
E_b Plate supply voltage (direct current).
E_c Grid bias voltage (direct current).
E_{gm} Maximum value of fundamental alternating-current grid voltage.
$_2E_{pm}$ Maximum value of double frequency alternating-current plate voltage.
E_g Instantaneous value of grid voltage.
E_p Instantaneous value of plate voltage.
$_0I_p$ Average value of direct plate current.
$_2I_{pm}$ Maximum value of double frequency alternating-current plate current.
I_g Average value of direct grid current.
I_1 Effective value of alternating-current grid current for measuring the alternating-current grid voltage.
I_2 Effective value of alternating-current tank current.
i_p Instantaneous value of plate current.
P_I Power input to the plate circuit.
P_o Power output from the plate circuit.
Eff.$_p$ Plate efficiency in per cent.
r External load resistance.
r_p Internal plate resistance.
r_2 Total tank circuit resistance.
wt_0 Angle of cut-off of plate current.
g_m Mutual conductance.
u Amplification factor.
L Inductance of tank circuit.
C Capacity of tank circuit.

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Volume 21, Number 1 *January, 1933*

MODULATION FREQUENCIES IN VISUAL TRANSMISSION*

By

Edwin Lee White

(Engineering Division, Federal Radio Commission, Washington, D. C.)

Summary—*A method of computing the maximum frequencies produced in television transmission is shown. This method is based on the consideration of the degree of edge definition produced. It is shown that these frequencies are independent of the amount of detail in transmitted pictures for equal edge definition.*

For a picture scanned by the usual method N pictures per second, with L lines, having a ratio r of width to height and having the ratio K between the width of scanning line and width of edge shadow, it is shown that

$$f = \frac{KNrL^2}{2}$$

Two systems varying from the normal are discussed and compared with the normal system in regard to the magnitude of the frequencies produced.

IN THE transmission of moving pictures by radio or wire one of the sources of distortion is frequency discrimination on the part of some portion of the equipment comprising the complete system both receiving and transmitting. This frequency discrimination may be due to many factors such as the aperture shape, the inclusion of too selective circuits, or the inability of the receiver light source to react with sufficient speed. It is not the purpose of the writer to discuss any of the many causes of distortion but to consider solely the frequencies which are required at the point of image production in order to produce a satisfactory image.

In current publications, articles with reference to visual broadcasting discuss the frequency bands occupied by various systems of picture transmission on the basis of the number of picture elements to be transmitted per second. This method of consideration is an outgrowth of the procedure used in the production of half-tone pictures in which each picture reproduced is actually divided into elements. Unfortunately, many individuals have concluded that the satisfactory transmission of simple images involves the emission of lower modulating frequencies than those involved in the transmission of complex images.

It is desired to approach the problem from a consideration of the conditions incident to the scanning of the edge of a black curtain against a white background such as represented in Fig. 1-A Were the visual system perfect the edge would be seen in the receiver exactly

* Decimal classification: R583. Original manuscript received by the Institute, June 30, 1932.

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as shown. However, due to various factors, the current in the reproducing device builds up gradually somewhat as shown in Fig. 1-B. As a result of this gradual building up of current the edge of the curtain in the received image is shaded as shown in Fig. 1-C. It is generally assumed that an edge shadow equal in width to the scanning line will be satisfactory. This assumption will be used initially in the following calculations:

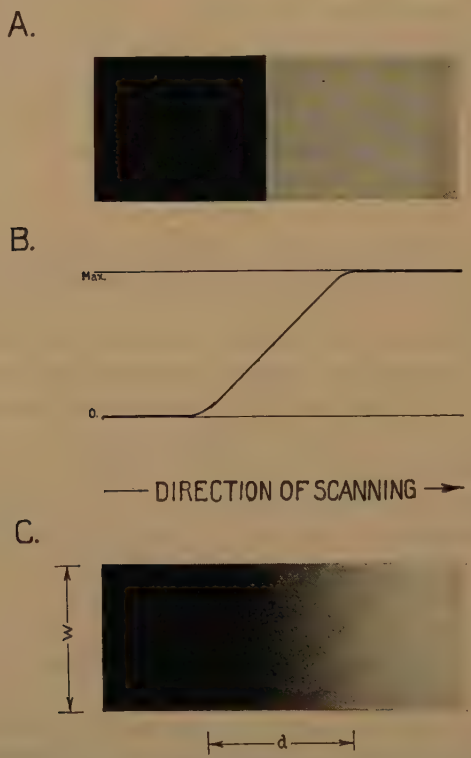

Fig. 1—A. The subject to be scanned. B. The current produced in the reproducing device. C. The image of the subject as reproduced.

The change in current shown in Fig. 1-B involves a half cycle of a frequency f, the magnitude of which is to be determined. This half-cycle change takes place during the movement of the scanning spot through the distance d during the time t (Fig. 1-C.).

$$f = \frac{1}{2t} \quad (t \text{ in seconds}) \tag{1}$$

The time t is to be determined.

As the edge shadow is reduced in magnitude the time t will approach 0 and the frequency generated will approach infinity.

If the picture is scanned N times per second, the time for scanning a single picture will be $1/N$ seconds. If L lines per picture are used the time for scanning a single line will then be $1/NL$ seconds.

Let r be the ratio of the width of the picture w to the height of the picture h

$$r = \frac{w}{h} \qquad h = \frac{w}{r}. \tag{2}$$

The width of a single line is the height of the picture h divided by L, and by definition is equal to d.

$$d = \frac{h}{L} \qquad h = \frac{d}{L}. \tag{3}$$

Combining (2) and (3)

$$dL = \frac{w}{r} \qquad \frac{d}{w} = \frac{1}{rL}. \tag{4}$$

The time t for scanning the distance d is to the time for scanning the length of a single line as the distance d is to the width of the picture w.

$$\frac{t}{\frac{1}{NL}} = \frac{d}{w}. \tag{5}$$

Combining (4) and (5)

$$\frac{t}{\frac{1}{NL}} = \frac{1}{rL}$$

$$t = \frac{1}{NrL^2}. \tag{6}$$

Substituting in (1)

$$f = \frac{NrL^2}{2}. \tag{7}$$

If the width of the edge shadow is to be limited to a width other than the width of a single line

$$f = \frac{KNrL^2}{2} \tag{8}$$

K being the ratio of the width of scanning line to the width of edge shadow.

It should be noted that the frequency computed above will appear each time the scanning beam encounters a junction between black and white, and is independent of the number of times such a junction appears. The same frequency appears in scanning a black curtain against a white background as appears in scanning a black grating against a similar background with equal edge definition requirements.

If a visual broadcast system has inadequate high-frequency response the edge of the black curtain mentioned above will be indefinite rather than sharp, and the grating will appear blurred or merely as an undefined shadow, depending on the degree of frequency discrimination.

Consideration of (7) and (8) also indicates that the modulation frequencies involved in visual broadcasting vary directly as the square of the number of lines per picture, as the number of pictures per second, and as the ratio between scanning line width and permitted edge shadow.

A typical example of expedients offered in the endeavor to reduce the frequency band width required for picture transmission is the proposal to transmit the top and bottom portions of a picture with a small number of lines and the central portion with a large number of lines. It is asserted that, psychologically, detail is needed only in the central portion of a picture for satisfactory appearance to the eye and, since the proposed system involves the transmission of a smaller number of "elements," a lower modulation frequency is required than would be involved were the whole picture scanned uniformly.

If the picture is to be divided into portions, each portion $1/a$ of the width of the whole picture, scanned with L' number of lines, and if each portion is to be scanned in $1/a$ of the time required to scan the whole picture, each portion may be considered as an individual picture transmitted in $1/aN$ seconds and having a ratio of width to height of ar. Substituting these values in (8)

$$f = \frac{KaNarL'^2}{2} = \frac{KNra^2L'^2}{2}. \tag{9}$$

If the whole picture were uniformly scanned with the same detail as the portion discussed above, the total number of lines required would be aL' and the maximum frequency produced would be, from substitution in (8)

$$f = \frac{KNr(aL')^2}{2} \tag{10}$$

which is identical with (9).

It is thus seen that the same frequencies are produced by this system as are produced by the normal system of scanning, given equal detail in the central portion.

Equations (9) and (10) are based on the assumption that the time occupied by scanning in great detail is proportional to the ratio between the height of the portion so scanned and the height of the whole picture. If this ratio is not retained the problem may be considered as follows:

Let a be the ratio between the time required to scan with detail and the time required to scan the whole picture and, let a' be the ratio between the height of the portion scanned in detail and the height of the whole picture.

Considering the portion scanned in detail as a single picture scanned in $1/aN$ seconds and having a ratio of height to width of $a'r$ and scanned with L'' lines, substitution in (8) gives

$$f = \frac{aNa'rL''^2}{2} . \qquad (11)$$

This equation differs from (9) solely in the difference between a^2 and the product aa'. The frequency produced by means of this second proposed system is to the frequency produced by uniform scanning, as computed above, as aa' is to a^2, which is equal to a'/a.

In conclusion, a method has been shown permitting the calculation of the order of modulation frequencies required for picture transmission based on the assumption of the relation between edge shadows and width of scanning line, which relation is the factor that limits the clarity and the maximum possible detail of a picture. The formula deduced does not differ materially from that obtained by the consideration of assumed picture elements.

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AN IMPROVEMENT IN VACUUM TUBE VOLTMETERS*

By

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Summary *—A vacuum tube voltmeter for the measurement of audio frequencies, which depends for its operation almost entirely upon the amplification factor of the tube, is described. This voltmeter combines the marked advantages of a single voltage source for filament, grid, and plate supply with the absence of zero adjustment requirements on the indicator. It has a further advantage over the more common type of vacuum tube voltmeters in that it can usually be made direct reading in volts on an 0-2 milliampere thermocouple type of indicating instrument.*

Introduction

WITHIN the past few years great forward strides have been made in the audio-frequency field. In keeping with modern practice it has been necessary to develop new instruments to maintain the higher standards of present-day equipment. Among the many instruments used, the thermionic tube voltmeter stands well up on the scale of importance. As a power level indicator (voltmeter), it is a permanent part of every audio-frequency installation of any size. Its importance in the laboratory, as well as in the field, has caused attention to be focused on its limitations in an effort to improve and simplify its operation.

Several years ago the need was felt for a vacuum tube voltmeter for audio-frequency measurements which would have a range of 0.05 to 3 volts, be reasonably portable, and be simple in operation. A study of the art was made. The slide-back voltmeter,[1] the straight rectifier type[2] (with and without amplifiers), and the push-pull[3] type of vacuum tube voltmeter were studied, tried, and discarded. The slide-back voltmeter, with three- or four-element tubes, often gives erroneous readings because the plate-current grid-voltage curve does not usually have a sharp intercept on the grid-voltage axis. The straight rectifier type, using a direct-current microammeter in the plate circuit, is subject to errors due to changes in the tube's plate impedance. These errors become large for low voltage measurements. The push-pull voltmeter requires two tubes and is subject to the same discrepancies as the rectifier type. All of the above instruments have large wave form errors.

* Decimal classification: R243.1. Original manuscript received by the Institute, July 17, 1932.

[1] R. A. Heising, U. S. Patent No. 1, 232, 919.
[2] R. H. Wilson, U. S. Patent No. 1, 287, 161.
[3] B. W. St. Clair, U. S. Patent No. 1, 857, 216.

It was soon realized that a new type of tube voltmeter would have to be developed if the requirements mentioned in the beginning of the previous paragraph were to be met. In order to lessen wave form error and to have a direct reading voltmeter, a tube operated as an amplifier and an alternating current indicating instrument were used. To simplify the adjustments a single source of direct-current voltage was employed.[4] If a vacuum tube voltmeter of this type is properly designed, the amplification factor of the tube may be made the controlling property. As this property depends almost entirely on the physical dimensions of the tube elements (and only to a small extent on the operating parameters) the use of this for the major controlling element is reasonable and sensible.

As the tube, used as an amplifier, may be operated in several different ways it is important to show why the circuits of Figs. 1 and 2 were employed in the final meters.

Voltage Amplifier

Theoretically, if a vacuum tube is to be used as a voltage amplifier operating under the conditions for maximum amplification, the external plate circuit impedance must be made infinitely high. Consider the equation for voltage amplification[5]

$$\mu' = \mu r_0/(r_p + r_0) \qquad (1)$$

where μ is the theoretical maximum of μ'; r_0 is the load resistance, and r_p is the plate impedance. In this equation μ' can equal μ only when r_0 becomes infinitely large compared to r_p. Under these conditions, not only would the amplification be large, but the value of μ' would become practically independent of the value of r_p. A tube then, used under these conditions, would theoretically make an ideal voltage measuring device. However, commercially available current measuring instruments which could be used in the plate circuit of a tube operated as above have internal resistances of the order of 800 to 1000 ohms. As the average tube's plate impedance is 20,000 to 30,000 ohms when operated on reasonably low voltages, its load impedance, in order to approach the condition for maximum amplification mentioned above, would have to be at least 100,000 ohms. It is obvious, therefore, that the indicating instrument could not furnish sufficient load impedance to operate the tube properly. Hence, it would be impractical to use the tube as a voltage measuring device.

[4] S. C. Hoare, U. S. Patent No. 1, 760, 597.

[5] See H. J. van der Bijl, "The Thermionic Vacuum Tube," p. 183, First Edition (1920), McGraw Hill Book Company.

Power Amplifier

If the tube is used employing a transformer in its output circuit the equation for the amplification (power) is[6]

$$\eta = \mu^2 r_g r_0/(r_p + Z_0)^2 \qquad (2)$$

where r_g is the input resistance and $r_0 + jx_0 = Z_0$ is the load impedance. This equation is a maximum when $Z_0 = r_p$. However, the amplification of the tube falls off slowly for larger values of Z_0, and the effect of changes in the tube's plate impedance on the total amplification becomes less, so it is best to arrange the plate load to be at least several times the plate impedance of the tube. As a transformer is used in the output circuit it becomes a relatively simple matter to arrange the

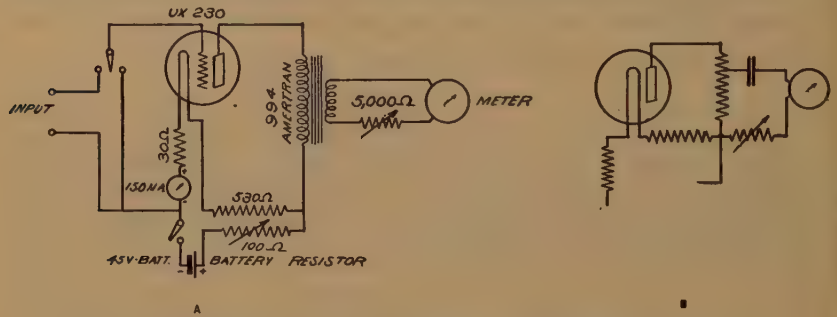

Fig. 1—(A) Circuit of one-tube vacuum tube voltmeter.
(B) Alternative output circuit for the voltmeter of Fig. 1A.

transformation ratio so that the reflected impedance in the primary is several times the tube's plate impedance.

Description of Apparatus

Several models of vacuum tube voltmeters employing the principles described above were constructed over a year and a half ago and are still giving excellent service. Fig. 1A shows a one-tube arrangement using a transformer in its plate circuit. Transformer coupling limits the frequency range of the instrument but increases the sensitivity. With a 2-milliampere thermocouple in the output the voltmeter has a range of 0.2 volt to 2.0 volts, a ten-to-one coverage. By using a high resistance voltage divider this range can easily be extended to 20 or more volts. All values are marked on the sketch. The direct-current source must be capable of supplying 60 milliamperes at from 40 to 48 volts. Two of the large radio "B" batteries connected in parallel are excellent

[6] *See* H. J. van der Bijl, "The Thermionic Vacuum Tube," p. 188, First Edition (1920), McGraw Hill Book Company.

for intermittent service. For continuous operation, however, a group of Edison type F2 storage cells would be better.

If a better frequency characteristic than that which is obtained from the voltmeter described is desired, the output network of Fig. 1B may be substituted in place of the one shown in Fig. 1A. This change, as mentioned previously, will of course greatly impair the sensitivity of the instrument. Recently, however, a 1-milliampere full-scale rectifier type of meter has been placed on the market. If this instrument is substituted for the 2-milliampere thermocouple shown in Fig. 1A, the sensitivity of the vacuum tube voltmeter using the network of Fig. 1B can be somewhat improved.

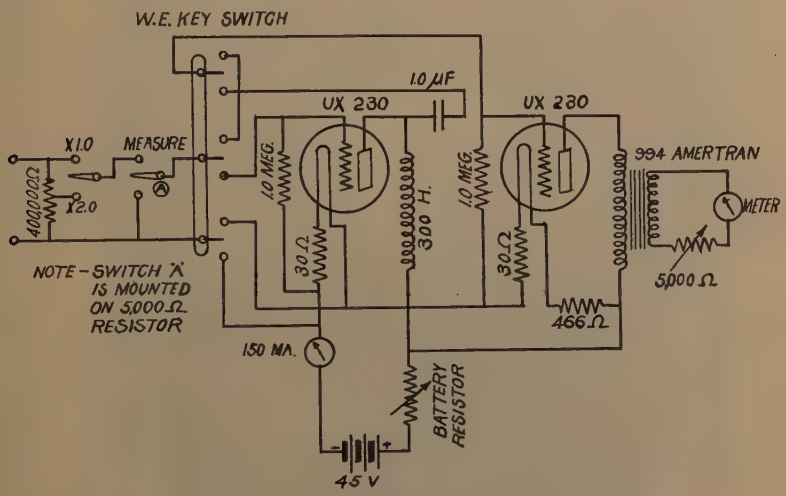

Fig. 2—Circuit of two-tube vacuum tube voltmeter.

In order to increase the range of the instrument, an amplifier tube was added. It was placed before the voltmeter tube in order to simplify switching. Fig. 2 shows the complete circuit of the vacuum tube voltmeter. The amplifier stage, as shown on the diagram, increases the range of the instrument by exactly ten times. With the two-to-one multipler shown on the input to the vacuum tube voltmeter, the range of this instrument is now from 0.02 volt to 4 volts.

Calibration and Accuracy

As was mentioned previously one of the most important characteristics of this instrument is its ease and premanency of calibration. If a proper output transformer is used, the indicating instrument may be of the dynamometer, copper-oxide rectifier, or thermocouple type.

With the vacuum tube voltmeter as shown in Figs. 1A and 2, both rectifier and thermocouple types of indicating instruments are used interchangeably. Because of the relatively high load impedance in the plate circuit and the use of the straight portion of the plate-current—

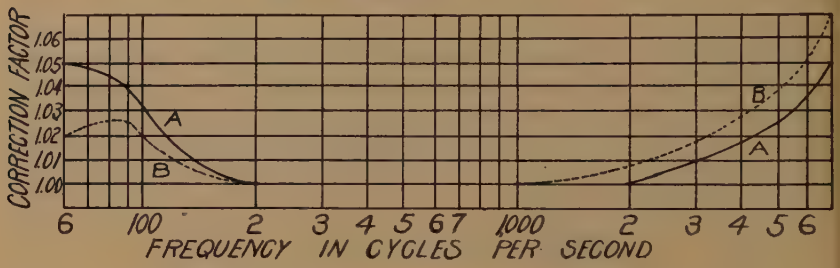

Fig. 3—Vacuum tube voltmeter correction curves. Solid line (curve A) for one tube; dotted line (curve B) for two tubes.

grid-voltage part of the characteristic, the actual method of calibration is simplicity itself. All that is necessary is to feed a known voltage into the input of the vacuum tube voltmeter and adjust the battery re-

Fig. 4—Panel view of the one-tube vacuum tube voltmeter. (Output meter not shown.)

sistor and/or an external resistor in series with the meter. These adjustments are carried out until the indicating instrument reads the true voltage directly on its scale. The only other check necessary is to determine the limiting voltages, that is, the voltages which will cause

the grid of the tube to swing positive, thereby lowering the input impedance of the device. This is easily done, as the plate-current—grid-voltage characteristic becomes curved at approximately this point, and therefore the indicating instrument will no longer directly follow the voltmeter across the input.

It is best to make this calibration on the straight portion of the vacuum tube voltmeter frequency characteristic curve. With the set-up as shown, a 1000-cycle calibration is usually used. If this one-point calibration is made often enough the accuracy of the instrument as a whole depends only on the accuracy of the indicating instrument used.

Fig. 5—Panel view of the two-tube vacuum tube voltmeter. (Output meter not shown.)

Using a Model 412 Weston thermocouple type of meter in the output circuit, the accuracy of the voltmeter is better than 1 per cent at all points on its scale. In a year and a half of intermittent use several of these voltmeters showed a change in calibration of less than 0.1 per cent. It might be interesting to note that a 13 per cent change in the current through the resistor system supplying the voltages for the tube causes only a 5 per cent change in the indicating instrument in the output circuit.

Frequency Characteristic

The vacuum tube voltmeter described was designed for use, in the audio-frequency range, (50 to 10,000 cycles per second). Its greatest use, however, has been between 50 and 6000 cycles per second, between which frequencies the greatest error is 5 per cent. As this is only ap-

proximately one-half decibel, it can usually be neglected. However, if greater accuracy is desired, the correction curves of Fig. 3 may be used.

Conclusion

In conclusion it might be well to summarize the advantages of this type of vacuum tube voltmeter.

(1) It is direct reading, actually measuring r-m-s values rather than peak average values.

(2) Calibration and checking of calibration is simple, as it only has to be done at one point on the scale.

(3) Permanency of calibration for one and one-half years is better than 0.1 per cent.

(4) As the frequency error is only 5 per cent between 50 and 6000 cycles per second usually no correction is necessary.

(5) The size of the instrument (one tube) is only 7 inches $\times$7 inches $\times$5 inches. This size could include a rectifier type of voltmeter for the output circuit, but would not include a thermocouple type of instrument similar to the Weston 412.

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TRANSOCEANIC RECEPTION OF HIGH-FREQUENCY TELEPHONE SIGNALS*

By

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Summary—*The application of high-frequency telephone transmission to international rebroadcasting is treated. Brief descriptions are given of the method used in rating the suitability of reception for rebroadcasting, the effects of magnetic disturbances upon transmission, the correlation of magnetic activity with transmission, and the forecasting of magnetic disturbances and resultant transmission conditions.*

WHEN the subject of international rebroadcasting was beginning to receive serious consideration, in 1928, a proposal was placed before the then-existing Board of Consulting Engineers of the National Broadcasting Company that observation reports of short-wave broadcast reception compiled by associated companies be forwarded to one point for correlation. It was believed that this would increase the value of the records and also indicate more clearly the possibilities and limitations of high-frequency transmission in the field of international broadcasting for it was obvious that the world-wide exchange of programs was destined to assume an important role in the future, and that high-frequency transmission would be the most effective medium for the interchange of programs. The National Broadcasting Company was requested to handle the correlation of these reports. Certain phases of that subject are presented in this paper. These phases are confined to the practical aspects of the investigation, and do not touch upon the many interesting theoretical considerations involved.

In addition to the reception reports received from the companies associated with the National Broadcasting Company at that time, (the RCA Communications Inc., the General Electric Company, and the Westinghouse Electric and Manufacturing Company), a considerable number of reception reports were also received in 1928 and 1929 from the British Broadcasting Corporation of England and Philips Radio of Holland, and in 1929 and 1930 from the Reichs-Rundfunk-Gesellschaft of Germany. These foreign reports covered reception primarily in those three countries of North American stations. The major circuits observed in the United States are indicated on the azimuthal chart of Fig. 1. Many of these circuits were observed simultaneously

* Decimal classification: R113. Original manuscript received by the Institute, July 19, 1932. Presented before New York meeting, June 1, 1932.

at Riverhead, N. Y., by the RCA Communications, Inc.; at Schenectady, N. Y., by the General Electric Company; and at Pittsburgh, Pa., by the Westinghouse Electric and Manufacturing Company. The telephone signals observed were mainly those of the services on the frequency bands assigned to international relay broadcasting. These frequencies are approximately 6, 9.5, 11.7, 15.1, 17.8, and 21.5 megacycles corresponding roughly to 50, 31, 25, 19, 17, and 14 meters. The distances involved were generally between 3000 and 5000 miles.

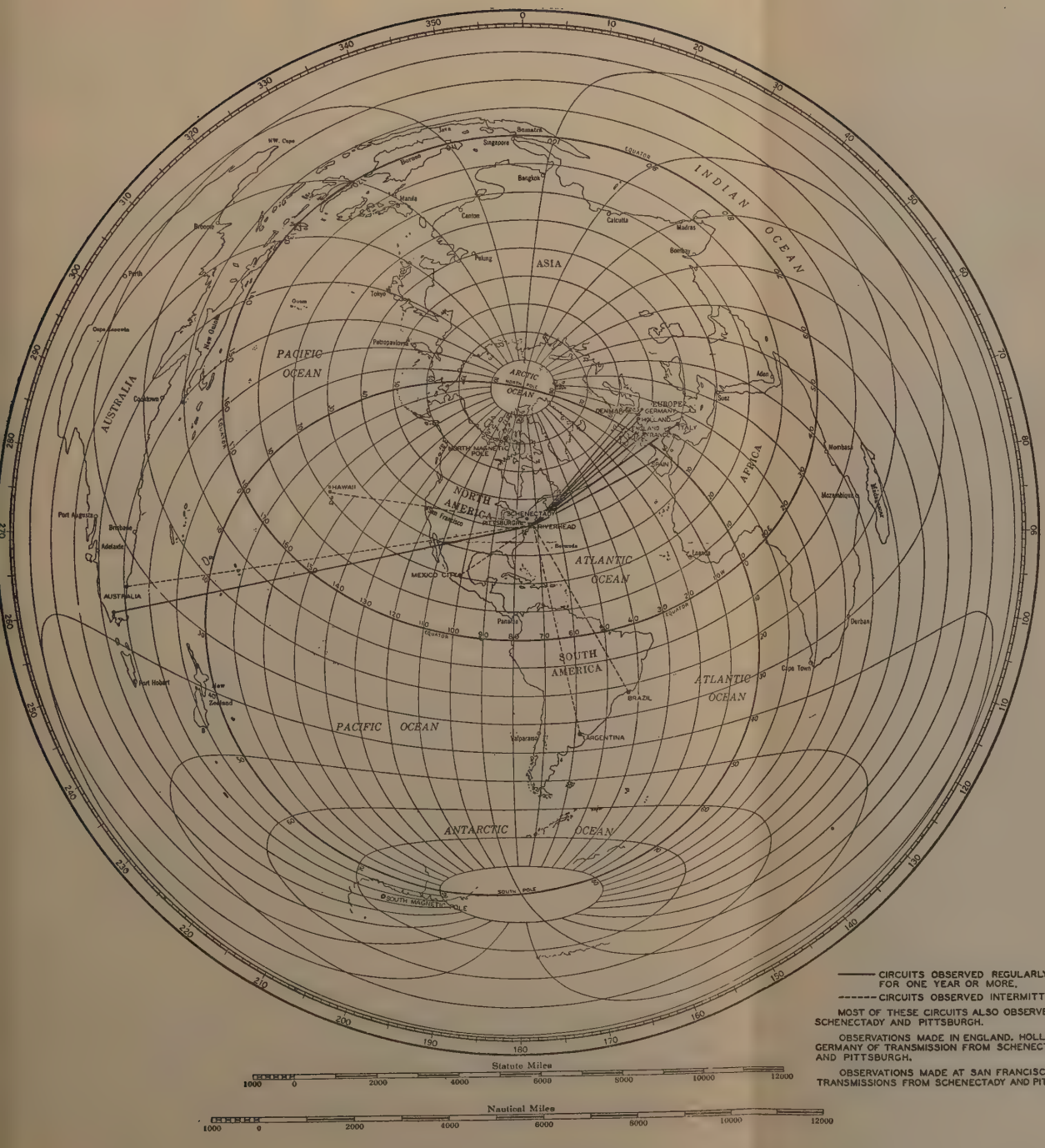
It was evident at the outset that a standard form of radio reception report was essential. The design of a reception log sufficiently simple and flexible to be used under varied conditions, and yet sufficiently comprehensive to indicate data quantitatively, presented a considerable problem. This was eventually solved satisfactorily by graphic representation of those factors which determine the quality of reception and the over-all value of reception for rebroadcast purposes. The assignment of numerical values for the over-all rating is probably the most outstanding feature of this log for although signal strength is of course very important, other factors, such as selective fading and interference, modify the usability of a signal to a very considerable degree. This log was adopted by all associated companies and by those foreign companies coöperating. Fig. 2 is a reproduction of an actual log, and it is evident that the signal characteristics are apparent at a glance. It was not until the adoption of this type of reception report that the work of correlation was placed upon a systematic basis.

Correlation of the reception reports was assumed to be capable of providing the following data:

(a) the diurnal limits of normal reception throughout the year, at specified geographical locations, of various short-wave broadcast stations on various frequencies,
(b) the amount of reception suitable for rebroadcasting,
(c) the degree of correlation existing between transmission and the major solar and terrestrial phenomena which may affect high-frequency propagation,
(d) the practicability of forecasting the occurrence of disturbing phenomena.

It was believed that if the attainment of these objectives could be achieved in, reasonable measure, international broadcasting would be placed upon a more practical basis, and program exchanges could be scheduled with a minimum of cancellations caused by failure of radio transmission circuits.

Although the limitations of high-frequency radio circuits were known in a general manner the requirements of high quality broadcast-



——— CIRCUITS OBSERVED REGULARLY FOR ONE YEAR OR MORE.

------ CIRCUITS OBSERVED INTERMITTENTLY.

MOST OF THESE CIRCUITS ALSO OBSERVED AT SCHENECTADY AND PITTSBURGH.

OBSERVATIONS MADE IN ENGLAND, HOLLAND, AND GERMANY OF TRANSMISSION FROM SCHENECTADY AND PITTSBURGH.

OBSERVATIONS MADE AT SAN FRANCISCO OF TRANSMISSIONS FROM SCHENECTADY AND PITTSBURGH.

Fig. 1—Circuits observed.

ing were so much more stringent than those of telegraphy or commercial telephony that the amount of data believed to be useful in the work of correlation was limited. The data available relative to terrestrial magnetic disturbances and high-frequency radio transmission in-

PLT.-16-1M-2-29

RADIO RECEPTION RECORD SHEET 279A
PLANT OPERATION AND ENGINEERING DEPT.
NATIONAL BROADCASTING CO. INC.

OBSERVER E.F.Bond WEATHER Clear Cold DATE Dec 24, 1929
RECEPTION AT Riverhead, L.I.

SINGLE FREQUENCY RECEPTION
STATION G5SW Chelmsford, England WAVELENGTH 25.53 M FREQUENCY 11,750 K.C.

MULTIPLE FREQUENCY RECEPTION – COMBINED OUTPUT

STATIONS	WAVE	FREQ.	NUMBER & DESCRIPTION RECEIVERS EACH FREQ.

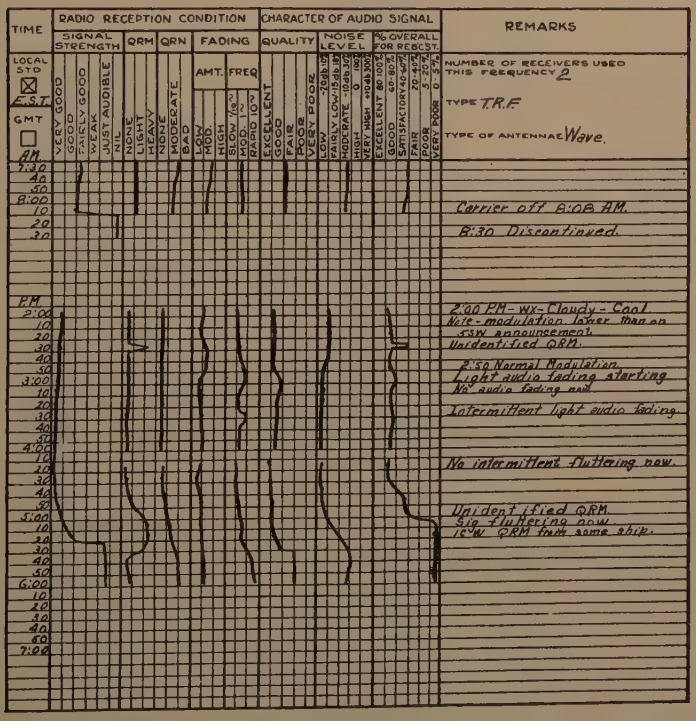


Fig. 2—Reception log.

dicated that transmission suffered severely during magnetic storms, and that these storms were more frequent and intense during years of maximum sun spots. Such information was, however, negative and pessimistic rather than of a nature furnishing useful information directly.

Determination of the normal diurnal and seasonal variations in transmission, dependent upon the degree of ionization existing over the path, was obviously impossible until the effects of magnetic disturbances could be recognized. At this point it should be noted that a disturbance of the earth's magnetic field is probably only one of a number of related transient phenomena resulting from or accompanying a solar disturbance, and that some of these, such as variations in the ionization and electron density of the upper atmosphere, are more effective in impairing high-frequency propagation than are the magnetic varia-

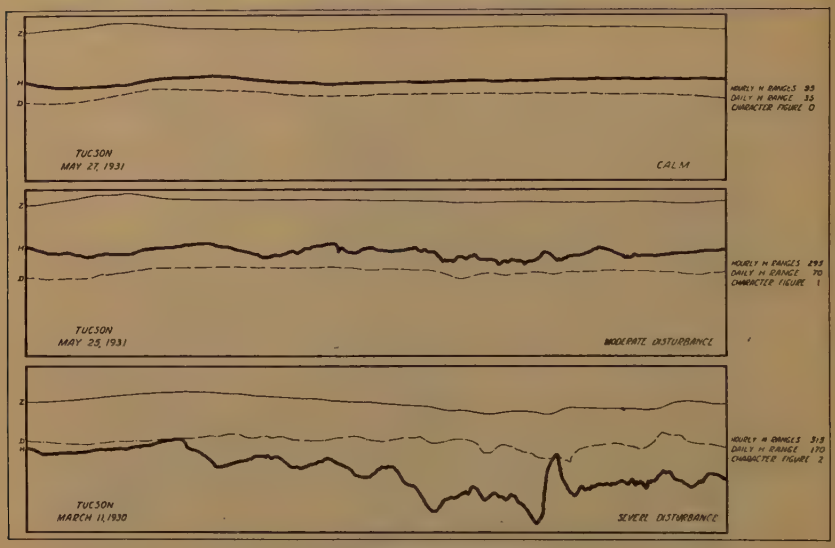


Fig. 3—Magnetograms.

tions. However, it is very convenient to speak of the effects of magnetic disturbances as the phenomenon of terrestrial magnetism is one of the few among those involved that permits of quantitative measurement.

Practically all magnetic observatories utilize recording equipment furnishing continuous graphic records of the declination (D) and the horizontal (H) and vertical (Z) components of the earth's field. These are called "magnetograms." Typical examples are shown in Fig. 3;—the first for a normal undisturbed day, the second for a day marked by a moderate storm, and the third for a severe storm.

Two methods are in general use for furnishing an immediate although approximate indication of the extent to which the earth's field is disturbed; daily character figures and daily ranges. The first method consists in assigning a figure to each day to designate its magnetic character, "0" indicating a calm or relatively undisturbed day, "1"

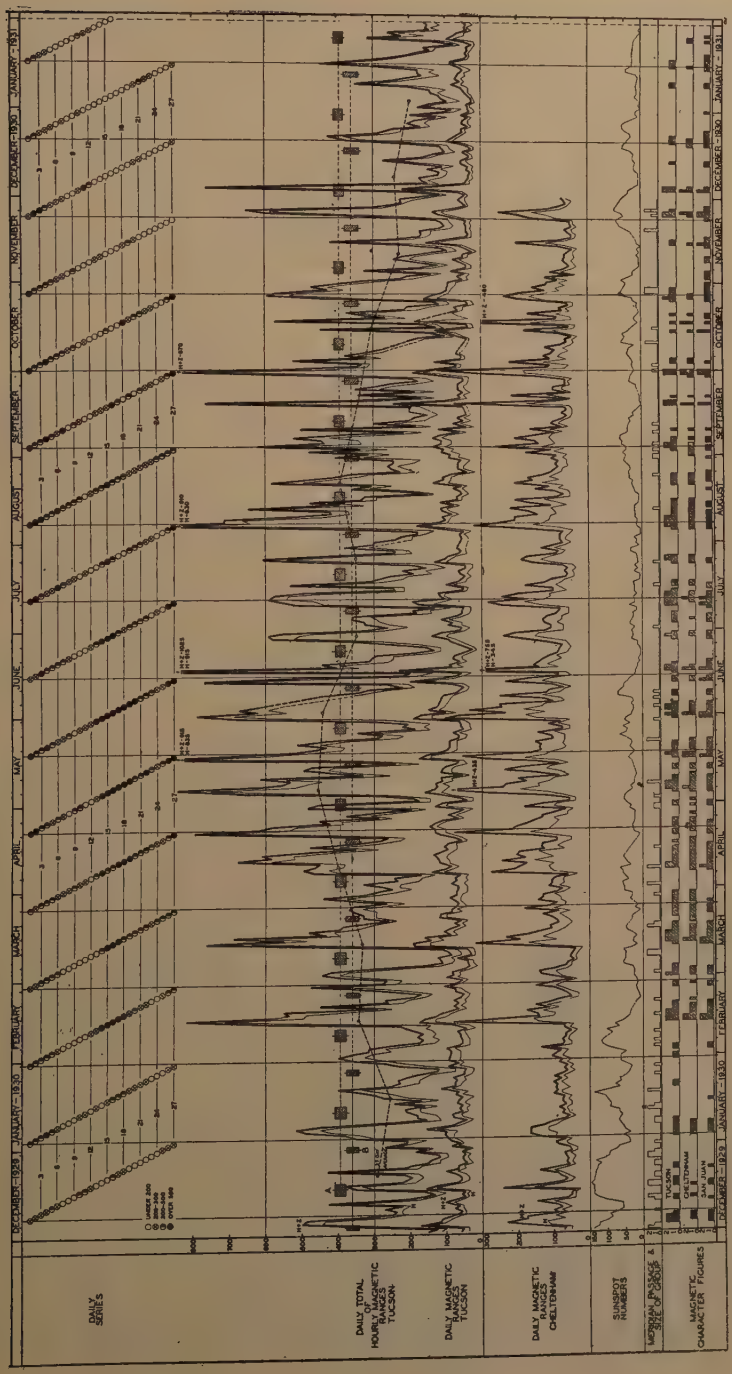


Fig. 4—Summation ranges and magnetic data—1930.

a day marked by a moderate disturbance, and "2" a day of severe disturbance. The second method generally utilizes the daily range between the maximum and minimum values of the declination or the *H*

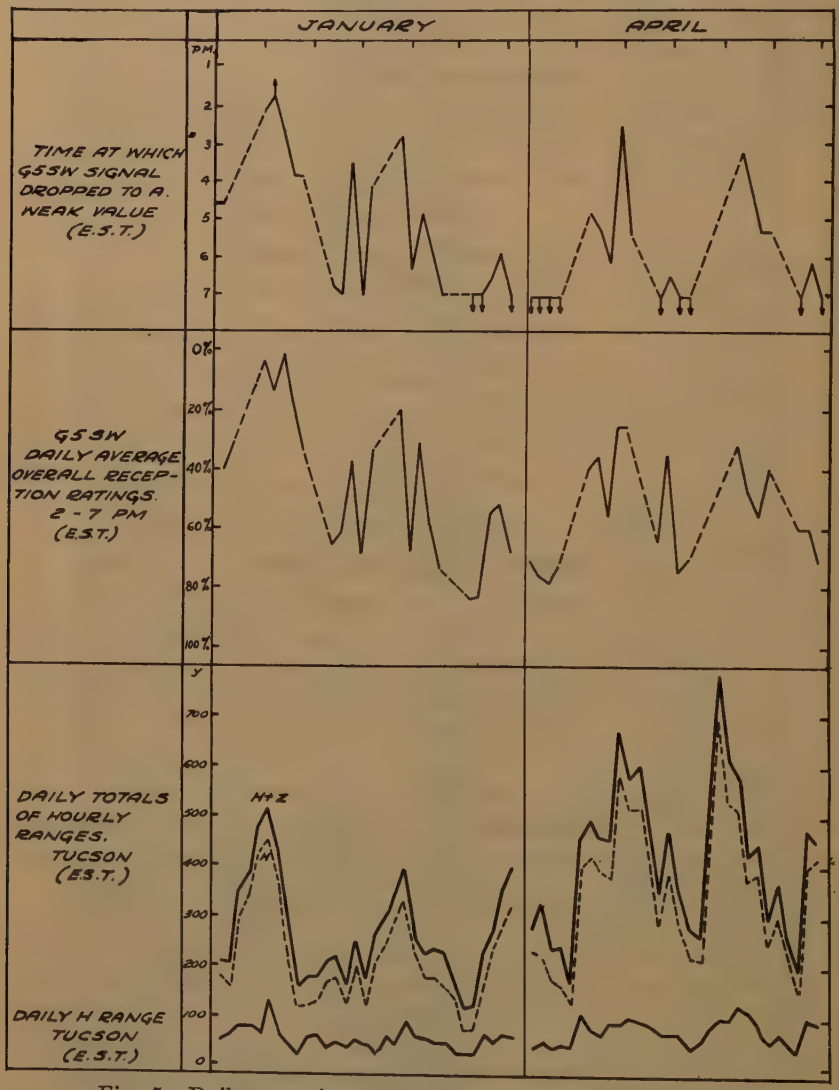


Fig. 5—Daily reception of G5SW January and April—1930.

component as an index. Neither of these indexes portray the intensity of a disturbance sufficiently to be of any material value for correlation with transmission. The character figures do not take into account

the large number of slight disturbances and are, in many cases, approximations. The daily ranges provide a somewhat better index, but it is readily apparent that disturbances of widely different degrees of intensity can exist within similar ranges. Various other methods, necessitating extensive computations, could have been used, but in view of the large amount of work involved were discarded as impractical, and a magnetic activity index was evolved, called the summation range, based upon the daily total of the hourly ranges of the horizontal component or the horizontal and vertical components together.

The use of this method over a period of years has proved its practicability as a very sensitive index. It is particularly effective in detecting the lesser disturbances which are not indicated by character figures or daily ranges but which affect short-wave transmission to a very considerable degree in winter or when frequencies close to the limiting values are used. Six-hour totals have been found very useful in this respect. The daily summation ranges for 1930 are shown in Fig. 4.

A very substantial degree of correlation between magnetic activity and transmission has been obtained by this method, and apparently transmission characteristics are very closely related to the degree of magnetic activity existing at the time. This is indicated in Fig. 5 which shows graphically, for Riverhead reception of G5SW, Chelmsford, England, during January and April, 1930, (a) the time at which the signal had decreased to a weak value, (b) the average over-all rating for the period 2–7 P.M., and (c) the daily total of the hourly H and Z ranges computed from the magnetograms of the observatory at Tucson, Arizona. January 15 furnished a typical example of a minor disturbance affecting transmission and readily detected by this method but which was not indicated by the character figures or daily ranges of three observatories.

Further correlation is indicated in Fig. 6 which shows data computed on a weekly basis through 1930 for the 2–7 P.M. daily reception of G5SW. This period, 2–7 P.M., is of particular interest because it marks the hours of the evening programs in Europe. The similarity between the curve of magnetic activity and that of the percentage of reception having ratings of good or higher is close, and weeks of high magnetic activity parallel weeks of low average reception ratings. An interesting feature of the magnetic curve is the recurrence of high and low values at intervals of four weeks. Despite the marked irregularity of the weekly reception the monthly percentage of reception suitable for rebroadcasting with over-all ratings of good or better remained practically constant at approximately 55 per cent for the first seven months and then decreased throughout the remainder of the year. This decline

was not due to an increase in the number or intensity of magnetic disturbances, for magnetic activity was lower during these months, but was the normal result of the autumn seasonal transition of this frequency which abruptly shifted the end of the useful reception period from the night hours to midafternoon.

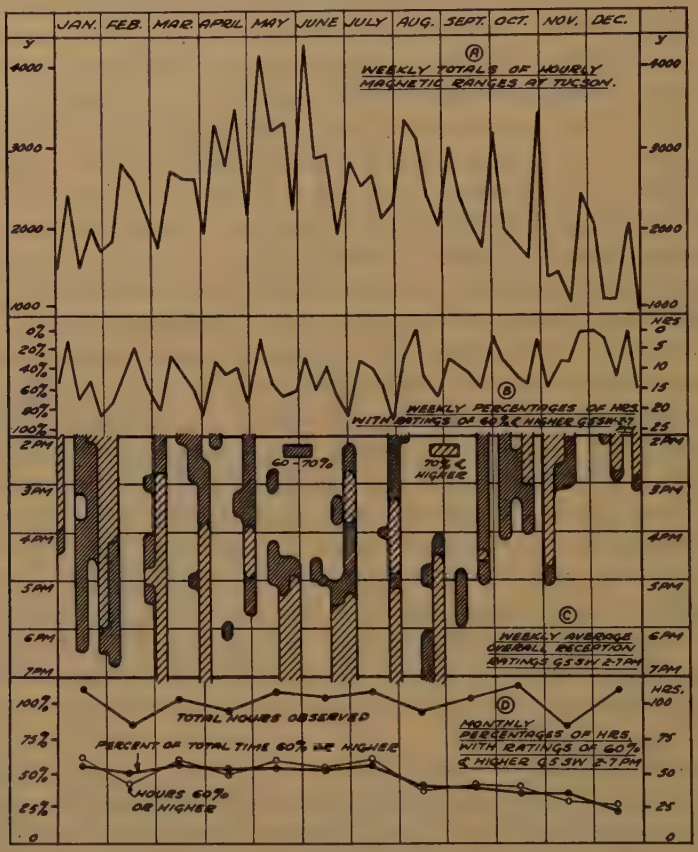

JAN. FEB. MAR. APRIL MAY JUNE JULY AUG. SEPT. OCT. NOV. DEC.
WEEKLY TOTALS OF HOURLY MAGNETIC RANGES AT TUCSON.
WEEKLY PERCENTAGES OF HRS. WITH RATINGS OF 60% & HIGHER G5SW 2-7PM
60-70%
70% & HIGHER
WEEKLY AVERAGE OVERALL RECEPTION RATINGS G5SW 2-7PM
TOTAL HOURS OBSERVED
PERCENT OF TOTAL TIME 60% OR HIGHER
HOURS 60% OR HIGHER
MONTHLY PERCENTAGES OF HRS. WITH RATINGS OF 60% & HIGHER G5SW 2-7PM

Fig. 6—Weekly average reception of G5SW—1930.

The diurnal limits of the useful period of G5SW on 11,750 kilocycles are shown in Fig. 7 for the year 1931. These limits are based upon the average over-all reception of the ten least disturbed days per month and, it is believed, are representative of normal reception. The general characteristics of these limits during the three years 1929–1931 have been very similar and offer encouragement that the methods of correlation used are adequate. Inspection of Fig. 7 indicates the existence of pronounced seasonal variations for 25-meter transmission from

Europe with the higher grades of reception extending over a period of hours centered around noon in winter, late afternoon in spring and fall, and evening in summer. In spring and fall, however, the reception limits were relatively unstable because of the pronounced changes resulting from the seasonal transitions occurring at approximately the time of the spring and fall equinoxes. The autumn transitions were

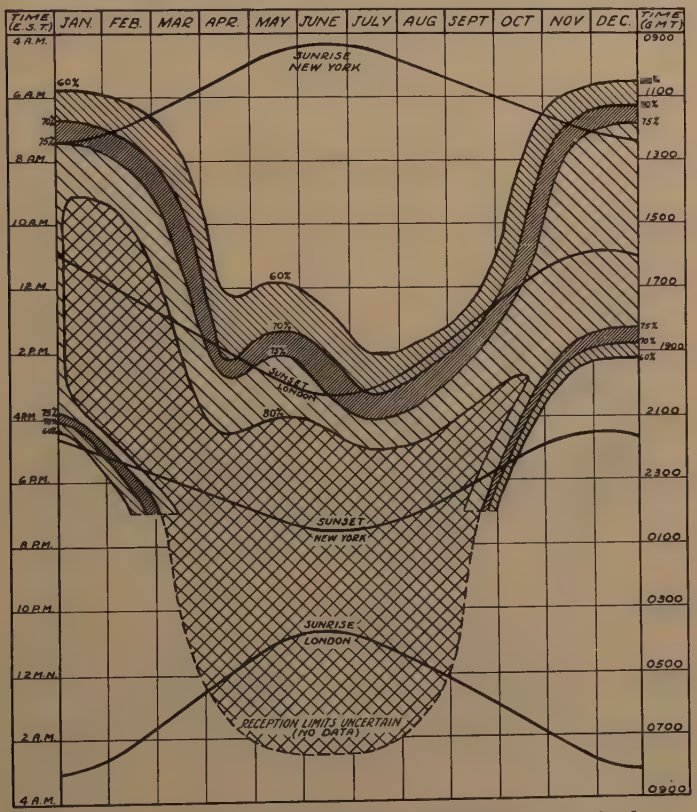


Fig. 7—Diurnal reception limits of G5SW Chelmsford, England 11,750 kc—1931.

usually very abrupt each year and carried the end of the useful period to an earlier hour than in the first part of the year. However, if the diurnal reception limits be based upon months starting at the winter solstice on December 21, instead of the calendar year, the chart becomes much more symmetrical.

The daylight-darkness distribution existing over the circuit did not appear to exert as pronounced an effect upon transmission of this frequency as upon some other frequencies although the greater proportion

of the higher grades of reception was obtained after the occurrence of sunset at the transmitter and the transmission path was partly or wholly in darkness. Sunrise and sunset dips were most pronounced during periods of low magnetic activity.

It is interesting to note that there has been a shift of the useful period to somewhat earlier hours each year, and that transmission con-

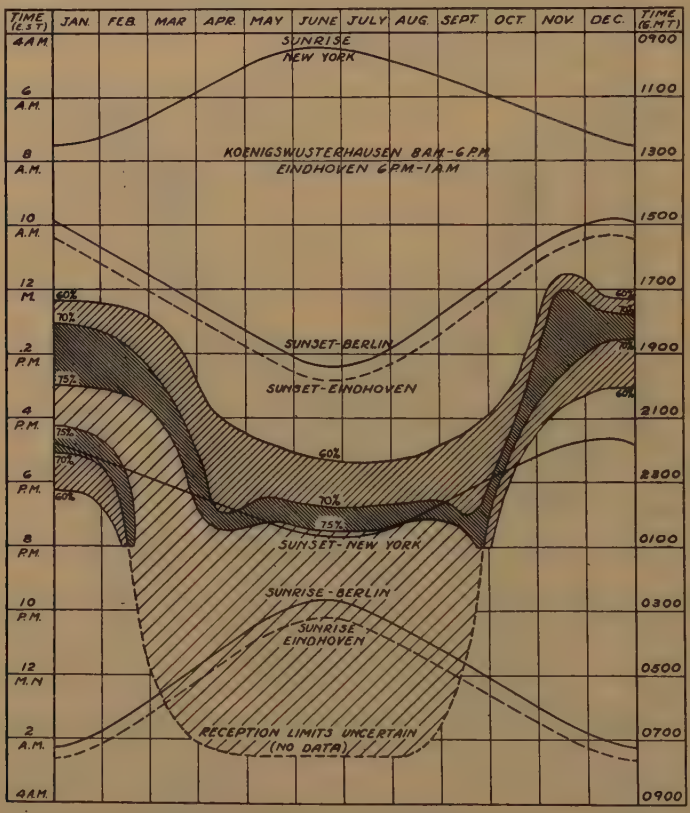


Fig. 8—Diurnal reception limits of PCJ Eindhoven, Holland, 9590 kc and Koenigswusterhausen, Germany, 9560 kc—1930.

ditions had improved so materially in 1931 over those of the two previous years that for the first time normal reception over the greater part of the year included ratings of excellent. Part of the increase in the amount of the higher grades of reception was of course due to improved receiver design; it is not likely to have been due to a lowering of the rating standard for rebroadcast purposes.

Similar charts have been prepared for reception of various other stations and other frequencies. All European circuits, however, showed

the same general characteristics. Fig. 8 shows the diurnal reception limits of the 31-meter transmissions of DJA, Koenigswusterhausen, Germany, and PCJ, Eindhoven, Holland. It is apparent that this frequency was of less value than 25 meters as the number of hours of good reception obtained was considerably less. This frequency was also more susceptible to magnetic disturbances, and the daylight-darkness dis-

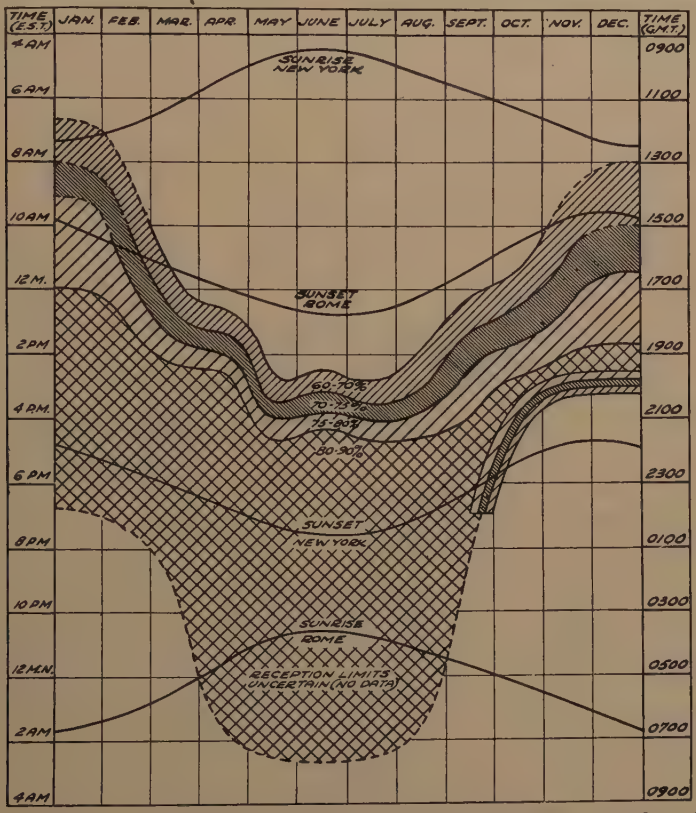


Fig. 9—Diurnal reception limits of I2RO Rome, Italy, 11,800 kc—1931.

tribution appeared to be a very important factor in propagation. Since there was little difference in the frequencies, locations, path lengths, and powers of these two transmitters an interesting opportunity existed to compare simultaneous reception. Over a period of weeks or months the average reception of these two stations was very similar although on some days there was considerable difference between the two signals.

This agreement of averages was also apparent in the simultaneous reception at Riverhead, Schenectady, and Pittsburgh of various Eu-

ropean transmitters. The greatest divergence was apparent during magnetic disturbances and at times when the signal strength was normally low. In general, however, if good reception of a foreign transmitter was obtained at one of these locations the reception at the other

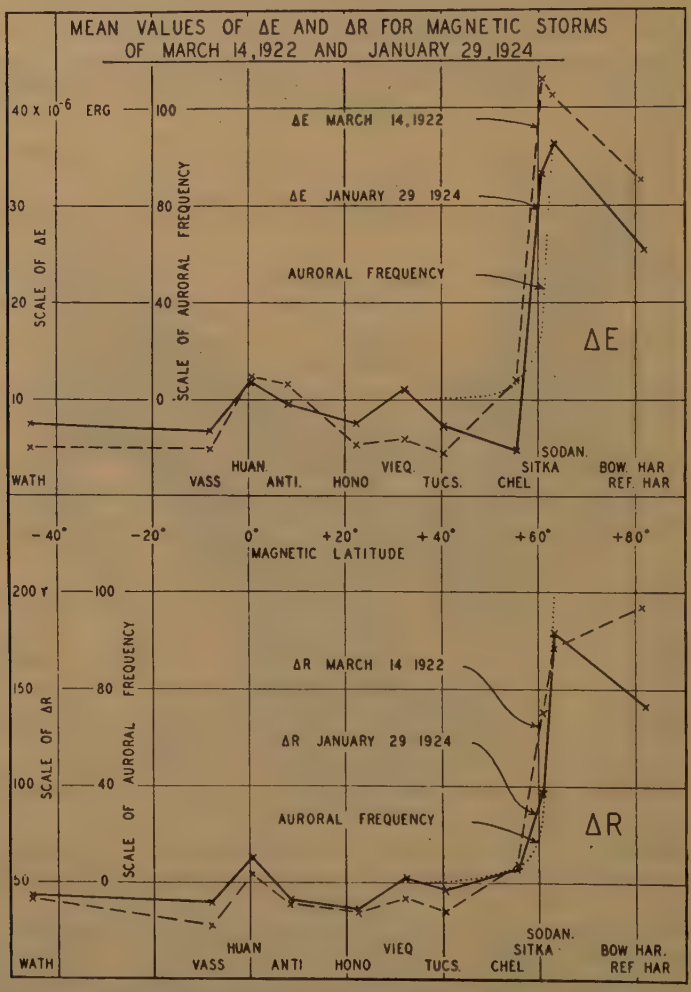


Fig. 10—Geographical distribution of magnetic disturbance. (Reprint *Journal Terrestrial Magnetism and Atmospheric Electricity.*)

two points was not greatly different. The only peculiarity noted in this three-point reception was that the build-up of the signals at Pittsburgh appeared to lag somewhat behind the build-up at the two more easterly locations.

The useful period for normal reception of I2RO, Rome, Italy, is shown in Fig. 9. This circuit, one of the most southerly of the European circuits observed regularly, was particularly interesting as transmission irregularities were much less severe during magnetic disturbances than on the more northern paths. Upon many occasions when signals on the northern circuits had faded out completely transmission on this circuit remained entirely normal, and it is evident that the southern European circuits possess a high degree of immunity to the detrimental effects of magnetic disturbances.

Similar immunity to many of the magnetic disturbances was apparent on the Pacific circuits and on north-south circuits such as those between North and South America. While the actual explanation is not readily apparent, it is probable that the geographical distribution of magnetic disturbances is an important factor and that the degree to which transmission is disturbed is dependent upon the magnetic latitude traversed by the circuit. Apparently the intensity of a magnetic storm is greatest at the auroral zone,[1] the lower limit of which over the North Atlantic is about 60 degrees north, and decreases quite rapidly south of this boundary. Fig. 10, which shows the severity of two of the so-called "great storms" as a function of magnetic latitude, indicates clearly the pronounced maximum in intensity at the auroral zone, and it is logical to assume that the minimum of interference on low latitude and north-south circuits is due in large measure to their distance from the area of maximum disturbance. This indicates the desirability of securing magnetic data from the high magnetic latitudes if possible. Data for this investigation, obtained primarily from the observatories at Cheltenham, Maryland, and Tucson, Arizona, which are considerably south of the auroral zone, have, however, provided a high degree of correlation with transmission over the North Atlantic circuits.

The effects of a magnetic disturbance usually became apparent a few hours after its commencement, and were evident as changes in transmission characteristics. Generally these changes could be placed in one or more of the following classifications:

(a) an advance in the normal daily trend of transmission which at times resulted in night transmission conditions existing during the day,

(b) increased fading which at the maximum produced a signal equivalent to one modulated with a low audio frequency,

(c) increased attenuation, which caused premature fade-outs of the

[1] W. F. Wallis, "Geographical distribution of magnetic disturbance," *Terr. Mag.*, vol. 36, p. 15–22, (1931).

signals and occasionally resulted in the complete disappearance of signals for considerable periods, amounting sometimes to days.

These effects were more or less progressive in the order indicated for the high latitude circuits, depending upon the rapidity with which the disturbance built up and its intensity, and were usually first apparent on the higher frequencies.

The trend advance consisted of a premature build-up of the signals, and was most pronounced on circuits traversing the high magnetic latitudes. This action progressed across the short-wave spectrum from the high to the low frequencies until a time was reached when the detrimental effects of the disturbance, increased fading and attenuation, interfered with the build-up and reversed the trend. In summer, particularly, the intensity of a disturbance often assumed considerable proportions before any detrimental effects were apparent, and as a result of the trend advance transmission very frequently showed a substantial improvement during the early stages of a storm, similar to the improvement produced by increasing darkness and a higher refracting layer over the transmission path. Premature fade-outs were almost invariably a positive indication of increased magnetic activity and conversely one of the most pronounced effects of quiet magnetic conditions was the length of the useful signal period.

The desirability of forecasting transmission conditions is obvious, and any method which will furnish even an approximate prediction of periods of disturbed and normal transmission, and the degree of disturbance which may be expected will be of considerable and immediate practical value from an operations viewpoint. The forecasting of transmission conditions, or strictly speaking the deviations of transmission from normal, depends upon the prediction of magnetic activity since this was the only phenomenon among those investigated, including sun spots, faculae and flocculi, that showed any day-by-day correlation with transmission. In turn, the prediction of magnetic activity must be based primarily upon its characteristics for although certain measures of magnetic activity, such as the diurnal variability, show a high degree of correlation with certain phases of solar activity, such as sun spots, when yearly or monthly averages are considered,[2] as indicated in Fig. 11, there does not appear to be any correlation for short periods of days or weeks.

Certain of these characteristics can be utilized very profitably in forecasting. The recurrence of disturbed and quiet magnetic conditions

[2] J. Bartels, "Terrestrial magnetic activity and its relations to solar phenomena," *Terr. Mag.*, vol. 37, p. 1–52, (1932).

at intervals of approximately 27 days is of course quite well known.[3] The importance of these recurrences, however, is that they constitute a life history of a storm. In many instances the primary disturbance which inaugurates a storm series is of comparatively low intensity and the cycle of build-up and decay, the former phase attended by increas-

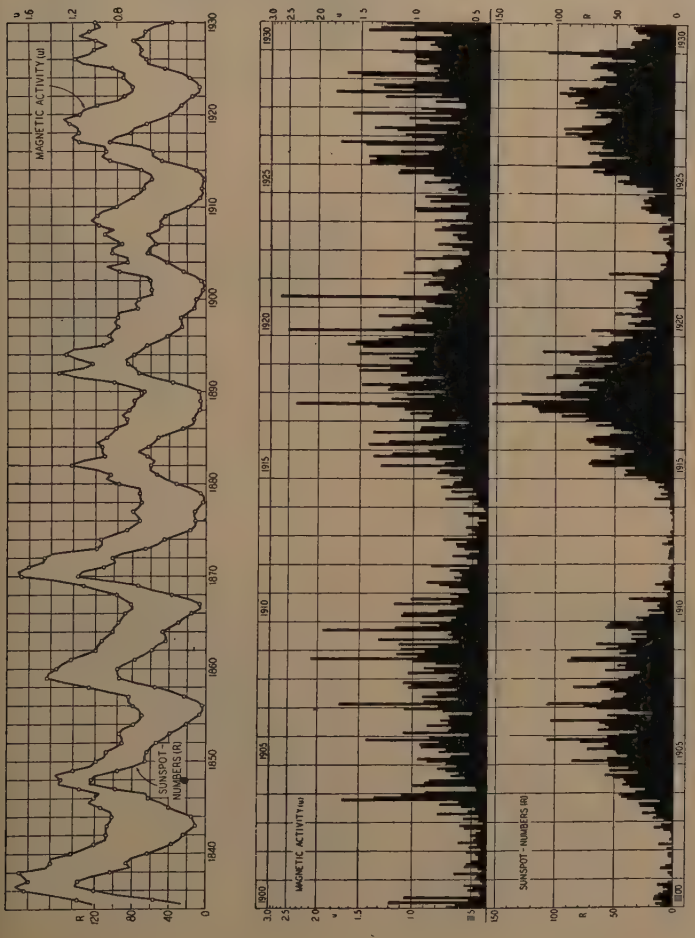


Fig. 11—Annual and monthly means of magnetic activity and sun spot numbers. (Reprint *Journal Terrestrial Magnetism and Atmospheric Electricity.*)

ing severity and extension of the disturbance and the latter phase by decreasing intensity and contraction, can be followed through successive recurrences. Sporadic disturbances, of which there are a considerable number and which at times are of great intensity, are impossible

[3] C. Chree and J. M. Stagg, "Recurrance phenomena in terrestrial magnetism," *Phil. Trans. Royal Soc.*, A, vol. 227, pp. 21–62, (1927).

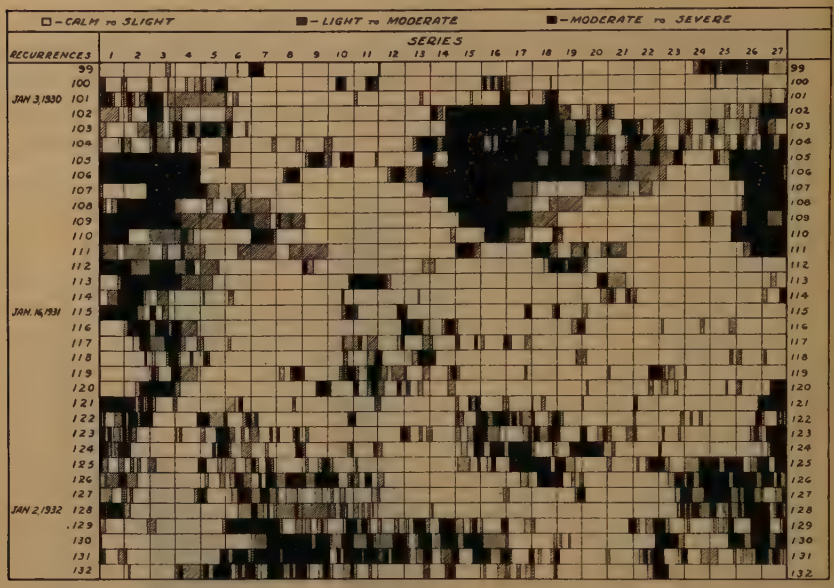


Fig. 12—Disturbance distribution.

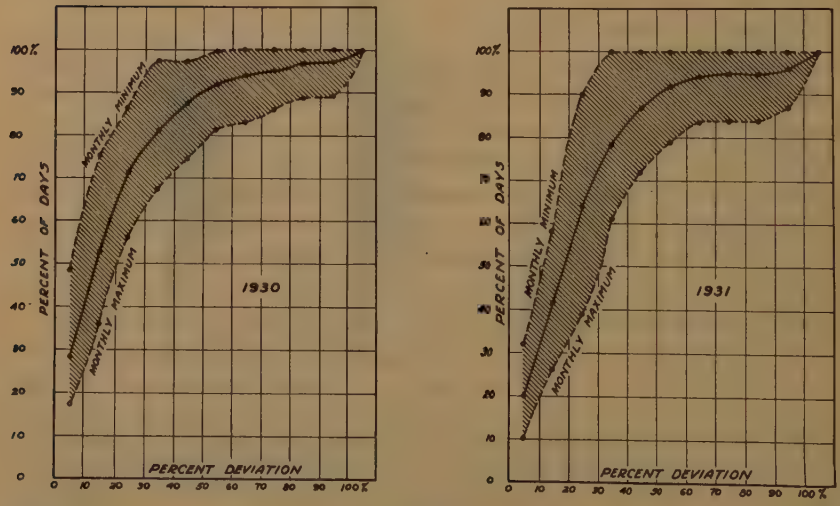


Fig. 13—Percentage deviation of predicted daily summation ranges, 1930–1931.

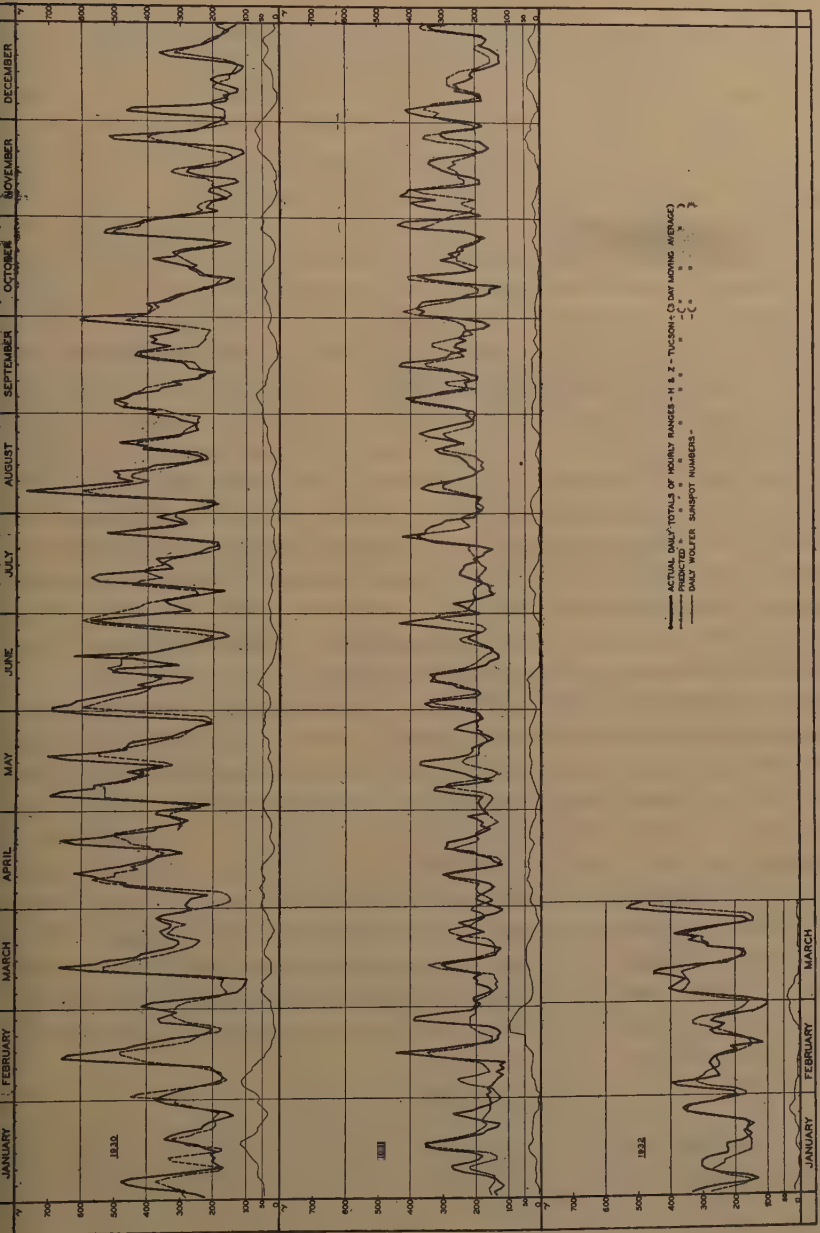


Fig. 14—Predicted versus actual daily summation ranges January, 1930, to March, 1932.

of prediction at the present time, and as the name implies seldom recur. Those of high intensity generally show certain characteristics which identify the disturbance as nonrecurring rather than the start of a storm series, while those of minor severity are usually confined within fairly definite storm periods. The disturbance distribution from November, 1929, to May, 1932, is shown in Fig. 12, based upon the Tucson magnetograms. Inspection of this chart indicates that although the major disturbances recurred at intervals of approximately 27 days the undisturbed periods showed a strong tendency to group themselves into extended sequences whose recurrences were either 26 or 28 days.

Predictions include the summation range in addition to the general trend and degree of disturbance, and sufficient accuracy has been attained in forecasting to warrant its inclusion as an operations function. In 1930 the deviations of the predicted summation ranges from the actual values were less than 20 per cent on 53 per cent of the days, and less than 30 per cent on 71 per cent of the days, and in 1931 the figures were 42 per cent and 64 per cent, respectively. These percentage deviations are shown graphically in Fig. 13. The highest degree of accuracy in predicting the occurrence and intensity of disturbances was attained during periods of greatest magnetic activity as indicated in Fig. 14 which shows the comparative predicted and actual summation ranges from January, 1930, to March, 1932.

The degree of deviation that can be tolerated before predicted transmission conditions change materially varies with the season, being about 25 per cent in winter and 50 per cent in summer, our observations having shown that the effects of magnetic disturbances upon transmission are greatest in the winter months. This is particularly important on single-frequency circuits and emphasizes the desirability of multifrequency circuits for successful international rebroadcasting. In addition to the use of several frequencies the reception reports have shown that transmitters should have a considerable amount of power, possibly as high as 50 kilowatts, when nondirectional antennas are utilized, if satisfactory and reliable relaying of broadcast programs is to be achieved.

Acknowledgment

The authors express their appreciation for the many courtesies extended to them by the United States Coast and Geodetic Survey.

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NORTH ATLANTIC SHIP-SHORE RADIOTELEPHONE TRANSMISSION DURING 1930 AND 1931*

By

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Summary—*Considerable data on radio transmission were collected during the years 1930 and 1931 incidental to the operation of a ship-shore radiotelephone service with several passenger ships operating in the North Atlantic. This paper discusses briefly the results of an analysis of these data. Contour diagrams are given which show the variation of signal fields with distance and time of day for the various seasons on approximate frequencies of 4, 9, 13, and 18 megacycles. Similar diagrams show the distributions of commercial circuits. Curves are also shown which enable the data to be applied more generally for other conditions of noise and radiated power.*

INCIDENTAL to the operation of a radiotelephone service between ships in the North Atlantic steamship lanes and the American Telephone and Telegraph Company's coastal stations at Ocean Gate and Forked River, N. J.,[1,2] an attempt has been made to collect as much data on radio transmission as possible. Records have been kept of how satisfactory the circuit was for commercial use on each contact and signal field strengths on radio transmission from ship to shore were obtained during most of the time. The study of these data involves a statistical analysis and because of the limitations in the amount of data available, the results are subject to modifications as the amount of accumulated data, over a period of years, increases.

Signal Field Strengths

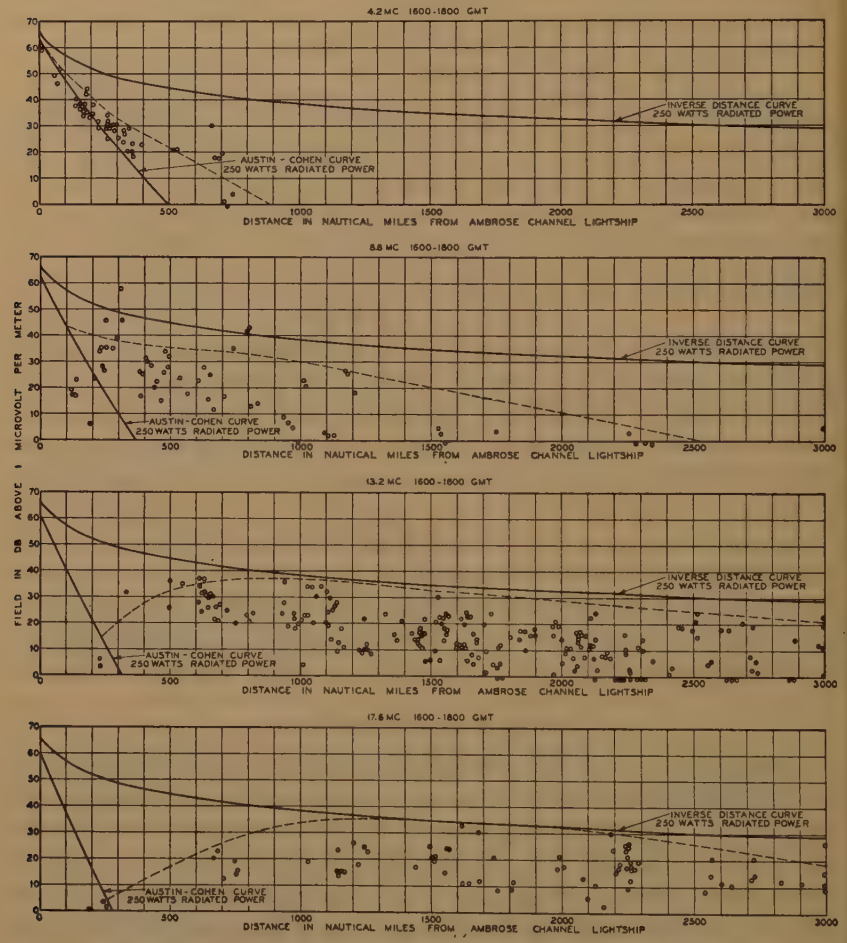
Frequencies in four radio-frequency bands are utilized for the ship-shore radiotelephone service; namely, approximately 4 megacycles, 8 megocycles, 13 megocycles, and 17 megocycles. Plots of the signal field strength data for the midday period 11 A.M. to 1 P.M. E.S.T. (1600–1800 G.M.T.) which have been obtained during the winter months, November, December, January, and February in 1930 and 1931, are shown as a function of distance in Fig. 1. Similar data for a late evening hour are shown in Fig. 2. The data as plotted are subject to the usual errors due to the measurement itself, determination of

* Decimal classification: R113×R270. Original manuscript received by the Institute, June 1, 1932. Presented before U.R.S.I., April ,29, 1932; presented before New York meeting, June 1, 1932.

[1] W. Wilson and L. Espenschied, "Radio telephone service to ships at sea," *Bell Sys. Tech. Jour.*, July, (1930).

[2] C. N. Anderson and I. E. Lattimer, "Operation of a ship-shore radio telephone system," Proc. I.R.E., vol. 20, p. 407–433; March, (1932).

distance, variations in radiated power, type of antenna used, and errors of analyses. A two-hour period is chosen to indicate the variation of the fields for daylight transmission as in the method of system operation, the shore station works first with ships in one half of the ocean


4.2 MC 1600-1800 GMT
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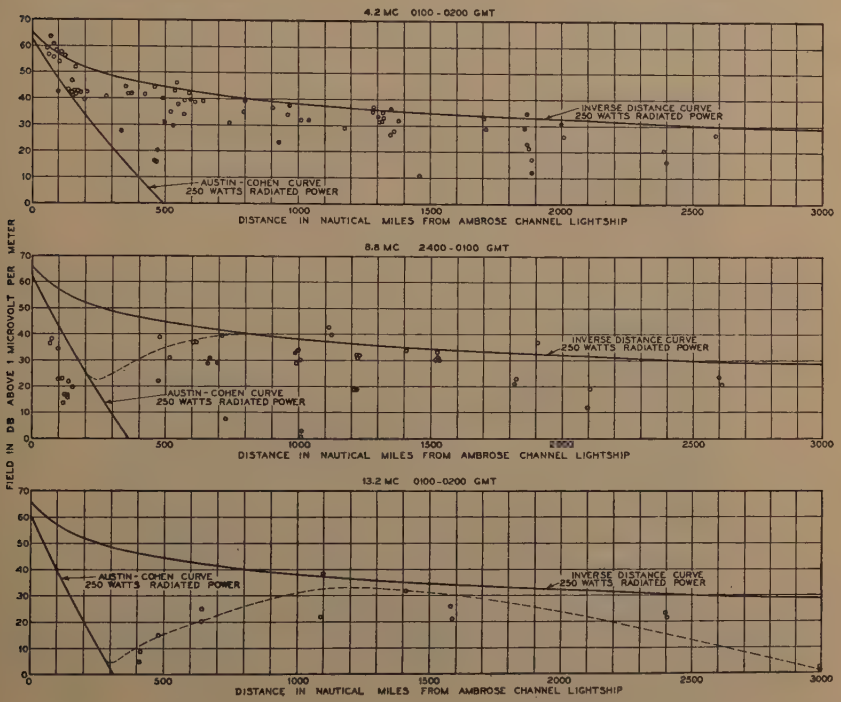
AUSTIN - COHEN CURVE 250 WATTS RADIATED POWER
DISTANCE IN NAUTICAL MILES FROM AMBROSE CHANNEL LIGHTSHIP
8.8 MC 1600-1800 GMT
FIELD IN DB ABOVE 1 MICROVOLT PER METER
INVERSE DISTANCE CURVE 250 WATTS RADIATED POWER
AUSTIN-COHEN CURVE 250 WATTS RADIATED POWER
DISTANCE IN NAUTICAL MILES FROM AMBROSE CHANNEL LIGHTSHIP
13.2 MC 1600-1800 GMT
INVERSE DISTANCE CURVE 250 WATTS RADIATED POWER
AUSTIN - COHEN CURVE 250 WATTS RADIATED POWER
DISTANCE IN NAUTICAL MILES FROM AMBROSE CHANNEL LIGHTSHIP
17.6 MC 1600-1800 GMT
INVERSE DISTANCE CURVE 250 WATTS RADIATED POWER
AUSTIN-COHEN CURVE 250 WATTS RADIATED POWER
DISTANCE IN NAUTICAL MILES FROM AMBROSE CHANNEL LIGHTSHIP

Fig. 1—Variation of radio field strengths with distance. Midday–Winter. Transmission from ships in north Atlantic steamship lanes to Forked River, New Jersey.

and then with ships in the other half so that data obtained during a one-hour period would not indicate the complete picture. The curves of inverse-distance and Austin-Cohen values corresponding to a radiated power of 250 watts are drawn in for reference. Each of the dotted curves indicates an approximation of the envelope of the maximum

fields. These curves are not determined by the data for that particular hour alone but also by the data for adjacent hours so that the set of curves indicates a continuous surface.

There are several reasons why the maximum values have been used instead of the average. First of all, inherent in the method of operation, data are usually obtained only when the circuit is known or thought to be satisfactory for commercial purposes. As soon as the circuit be-


4.2 MC 0100 - 0200 GMT
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AUSTIN - COHEN CURVE 250 WATTS RADIATED POWER
DISTANCE IN NAUTICAL MILES FROM AMBROSE CHANNEL LIGHTSHIP
8.8 MC 2400 - 0100 GMT
INVERSE DISTANCE CURVE 250 WATTS RADIATED POWER
AUSTIN - COHEN CURVE 250 WATTS RADIATED POWER
DISTANCE IN NAUTICAL MILES FROM AMBROSE CHANNEL LIGHTSHIP
13.2 MC 0100 - 0200 GMT
AUSTIN - COHEN CURVE 250 WATTS RADIATED POWER
INVERSE DISTANCE CURVE 250 WATTS RADIATED POWER
DISTANCE IN NAUTICAL MILES FROM AMBROSE CHANNEL LIGHTSHIP
FIELD IN DB ABOVE 1 MICROVOLT PER METER

Fig. 2—Variation of radio field strength with distance. Nighttime–winter. Transmission from ships in north Atlantic steamship lanes to Forked River, New Jersey.

comes uncommercial, a shift is made to another frequency. This tends to make the average appear higher than it really should be. Secondly, the number of data at any particular distance for a given hour is usually too small to obtain a satisfactory average. To get sufficient points the units of time and distance would have to be increased, often to the extent of the mean losing its significance. Lastly, it is felt that the envelope of maximum values has a physical significance in that studies previously reported on have tended to show that the inverse-distance values represent a practical maximum limit of field strengths for radio

transmission over sea water. The minimum values, approximated by the Austin-Cohen formula, are essentially limited to strictly ground wave transmission and, except for transmission over relatively short distances, usually lie below the lower limit of measurement.

The field strength values below the Austin-Cohen curve on the 8-megacycle and higher frequencies might be explained by assuming less radiated power along the ground due to the directive properties of the transmitting and receiving antennas at these frequencies.

The distance range at which inverse-distance values may be obtained on a given frequency as well as the time of day may be very

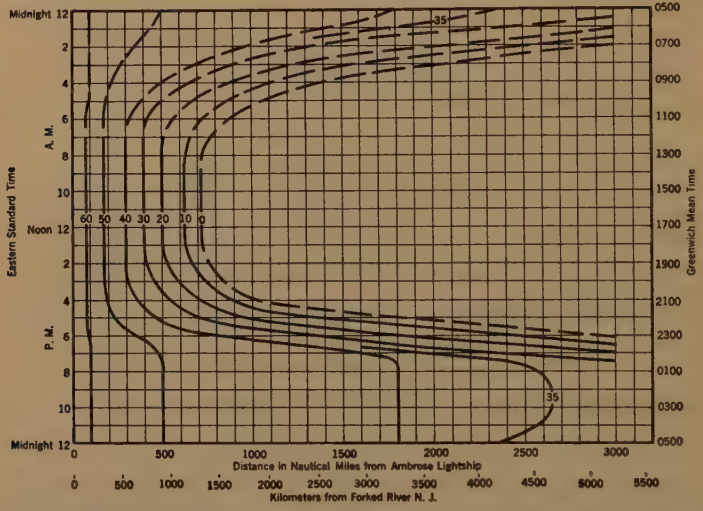

Fig. 3—Approximate variation of radio field strengths with distance and time of day. Contour lines indicate maximum expected field strength in decibels above 1 microvolt per meter. Frequency 4.2 mc. Winter, 1930–1931. Corrected to 1-kw radiated power. Solid lines indicate portions of contours determined by ship-shore data.

much limited. Then too, the degree to which the field strengths approach the maximum values depends in part upon the absence of any unusual transmission conditions such as occur during periods of solar disturbances. From the dashed curves of Figs. 1 and 2 and similar curves for the other hours of the day, field strength surfaces can be built up. The field strength contours of such surfaces for the four frequencies, 4.2 megacycles, 8.8 megacycles, 13.2 megacycles, and 17.6 megacycles for the winter months, are shown in Figs. 3 to 6, inclusive. Similar contour patterns for summer and for the spring and fall months are included in the appendix.

As the field strength data were obtained for the most part during the period 7 A.M. to midnight E.S.T. (1200 to 0500 G.M.T.), the parts of

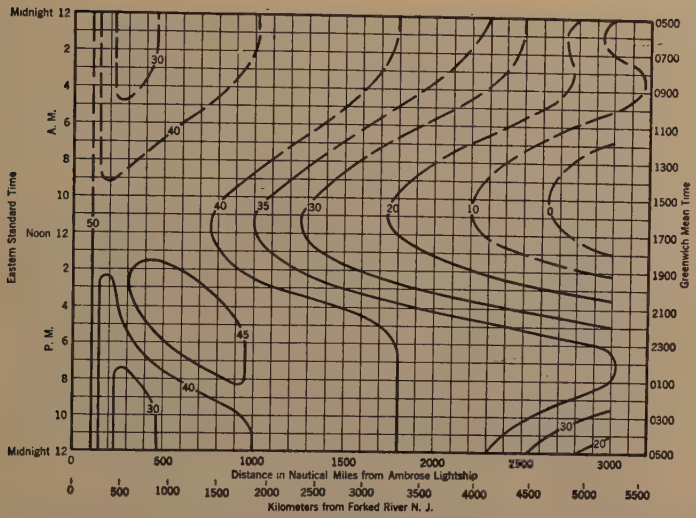

Fig. 4—Approximate variation of radio field strengths with distance and time of day. Contour lines indicate maximum expected field strengths in decibels above 1 microvolt per meter. Frequency 8.8 mc. Winter, 1930–1931. Corrected to 1-kw radiated power. Solid lines indicate portions of contours determined by ship-shore data. Information on diurnal characteristic at 3000 miles supplemented by point-to-point transmission.

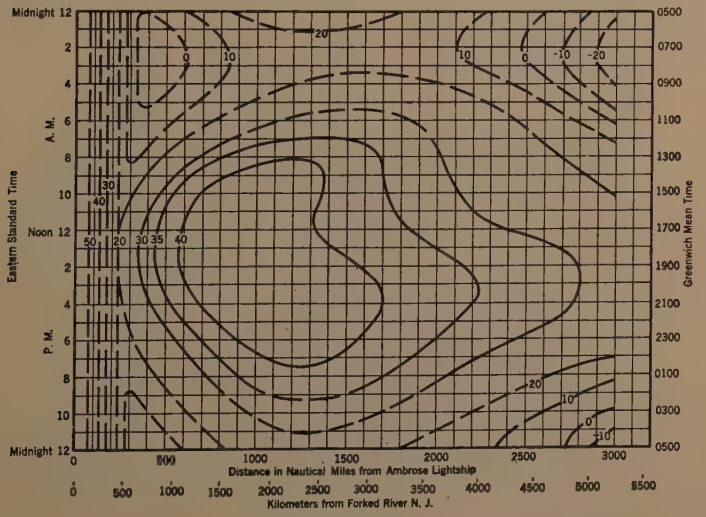

Fig. 5—Approximate variation of radio field strengths with distance and time of day. Contour lines indicate maximum expected field strengths in decibels above 1 microvolt per meter. Frequency 13.2 mc. Winter, 1930–1931. Corrected to 1-kw radiated power. Solid lines indicate portions of contours determined by ship-shore data. Information on diurnal characteristic at 3000 miles supplemented by point-to-point transmission.

the curves determined by extrapolations and interpolations are shown by the dashed lines. In addition to the data obtained in connection

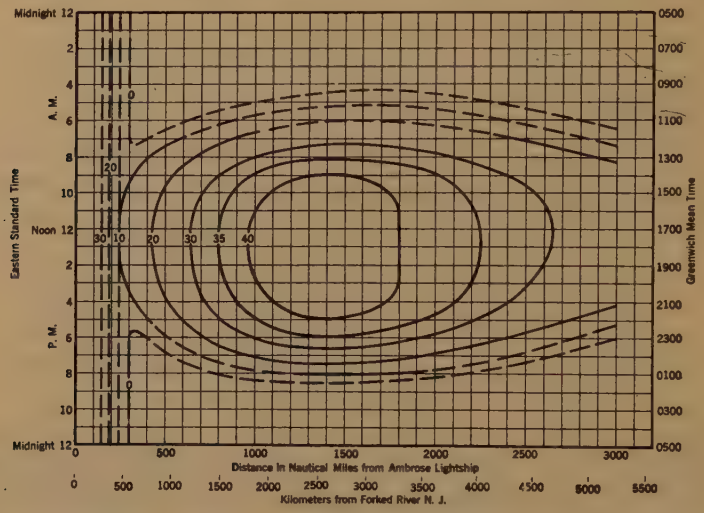

Fig. 6—Approximate variation of radio field strengths with distance and time of day. Contour lines indicate maximum expected field strengths in decibels above 1 microvolt per meter. Frequency 17.6 mc. Winter, 1930 and 1931. Corrected to 1-kw radiated power. Solid lines indicate portions of contours determined by ship-shore data. Information on diurnal characteristic at 3000 miles supplemented by point-to-point transmission.

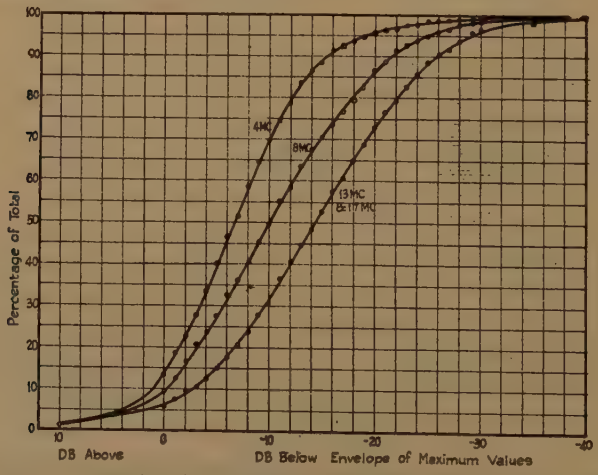

Fig. 7—Distribution of measured fields with respect to envelope of maximum values. 1930–1931.

with ship-shore operation, use was made of data obtained in transatlantic point-to-point operation between Rubgy, England, and Netcong,

N. J., on frequencies approximately 9 megacycles, 13 megacycles, and 18 megacycles. These data, as well as experimental data obtained by the Bell Telephone Laboratories,[3] were particularly helpful in determining the position of the contours at the 3000-mile distance for most of the 24 hours.

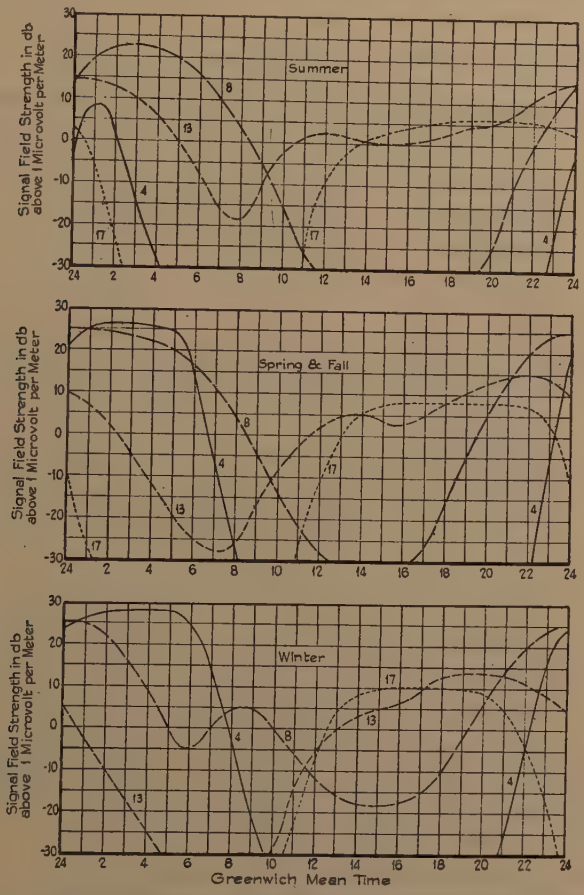

Fig. 8—Diurnal variation of signal field strengths. Estimated mean values derived from idealized curves. Corrected for 1-kw radiated power of signal transmission. Distance 3000 nautical miles—north Atlantic transmission. Figures indicate approximate frequencies in megacycles. 1930–1931.

The fields indicated by the contour curves of Figs. 3 to 6, inclusive, representing as they do the maximum values, are obtained comparatively seldom. The distributions of the measured values with respect to these curves, as well as the similar curves in the appendix, are shown

[3] C. R. Burrows, "The propagation of short radio waves over the north Atlantic," Proc., I.R.E., vol. 19, p. 1634–1659; September, (1931).

in Fig. 7. The distributions differ with frequency probably because the higher frequencies are used more generally for the greater distances resulting in less stable transmission paths. The mean for 4.2-megacycle transmission, above and below of which we find 50 per cent of the values, is 7 db below the envelope of maximum values. For 8.8 megacycles the mean is 10 db below and for 13.2 megacycles and 17.6 megacycles it is 15 db below. The resultant means of fields on the dif-

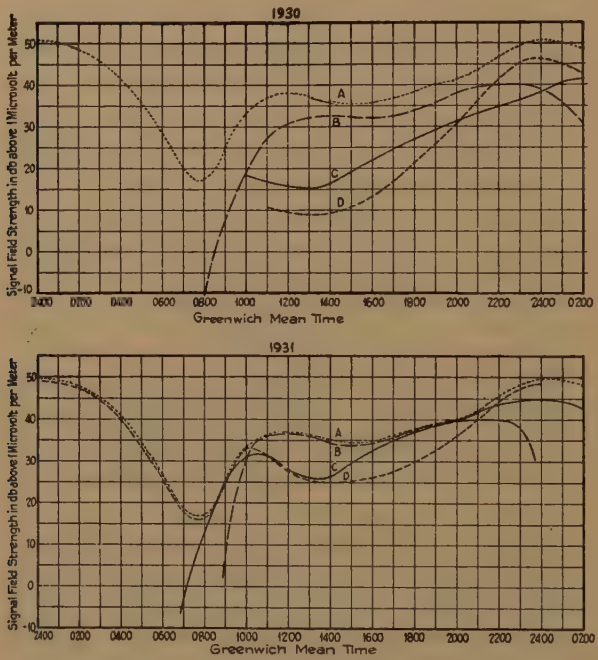

Fig. 9—Variation of maximum 13 mc signal field strengths. Diurnal variation for different transmitting stations; Rugby, England, to Netcong, New Jersey. May, June, July, August.
A—Envelope of maximum values for all data.
B—14,440-kc transmission—"TW" array
C—12,290-kc transmission—"Sterba" array
D—12,150-kc transmission—"Pine-tree" array

ferent frequencies for a distance of 3000 nautical miles for the various seasons are shown in Fig. 8. These curves can be obtained from the field strength contour diagrams and the distribution curve of Fig. 7.

The part played by the transmitting and receiving antennas in determining the radio-transmission characteristic is apt to be considerable as illustrated by Fig. 9. This figure shows the envelope of maximum values obtained in transmission from England to the United States on frequencies in the general vicinity of 13 megacycles with

different types of antennas for the summers of 1930 and 1931. The characteristics differ considerably and although, of course, it is hard to differentiate between the antenna factor and the effect of the slight frequency differences, it does give some idea of the variation one is likely to encounter with relatively small differences in the conditions. The dotted curve corresponds to the contour lines at 3000 miles of the contour pattern showing the diurnal variation of 13-megacycle signal fields for summer.

Of interest is the comparison between the curves for 1930 and 1931. The 1931 increase in curve *B*, which was already near the assumed maximum *A*, is much smaller than in curves *C* and *D*.

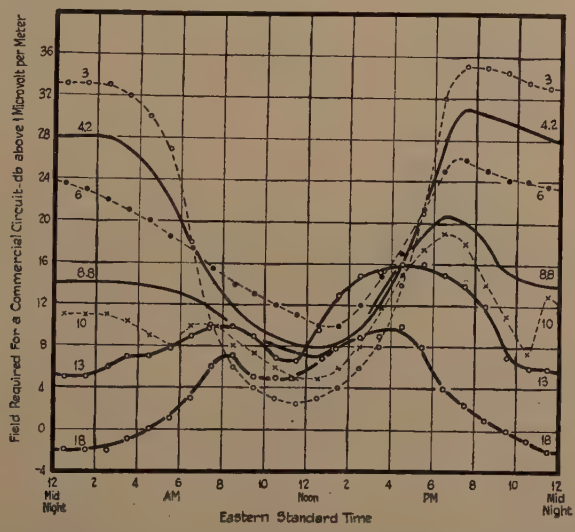

Fig. 10—Diurnal variation of radio noise measured in terms of signal field strength required for a commercial circuit. Simple vertical receiving antenna. Forked River, New Jersey—spring and fall months. Figures denote approximate frequency in megacycles.

Distribution of Commercial Time

The two most important factors in determining the effectiveness of a radio circuit are the signal field strength and noise. In addition there are factors which affect the signal-to-noise ratios such as receiving antenna discrimination against noise, receiving antenna directivity in both the horizontal and vertical planes and the relation of these patterns to the direction of the received signal, and the directivity of the transmitting antenna. Fading becomes increasingly important as the distance and the transmitting frequency increase.

The diurnal variation in radio noise is shown in Fig. 10 which gives

the field strength of a steady signal required to give a circuit just usable for commercial telephone communication. The diurnal variation is greatest on the lower radio frequencies so that, whereas 8- to 10-decibel signal fields are satisfactory for daytime reception of 4 megacycles, 28- to 30-decibel signal fields are required at night. The curves for 4.2 megacycles and 8.8 megacycles are interpolated from the measurements on frequencies on either side.

The approximate variation of percentages of usable circuits with distance and time of day on the various frequencies for the winter

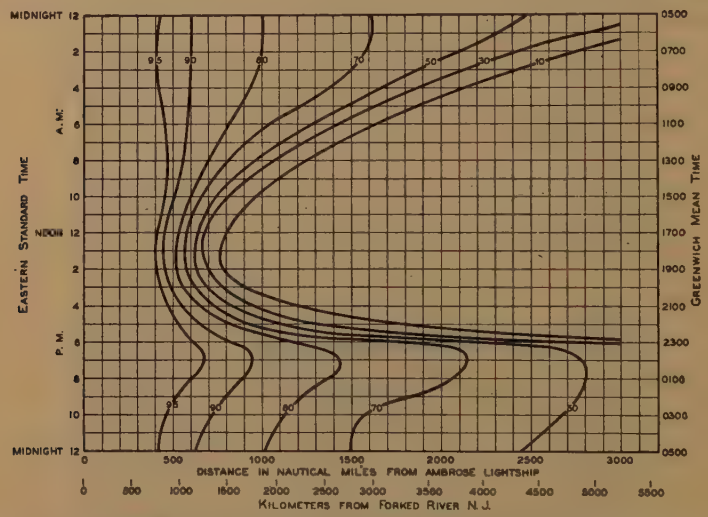

Fig. 11—Approximate variation of per cent commercial time with distance and time of day derived from idealized curves of signal field, data of diurnal variation of radio noise, and data on commercial circuits correlated with signal-to-noise ratios. Transmission from ships in north Atlantic shipping lanes to Forked River, New Jersey. Radiated power approximately 250 watts. Receiving antenna arrays used. Frequency 4.2 mc. Winter, 1930–1931.

months are shown in Figs. 11 to 14, inclusive. Similar curves for summer and spring and fall are given in the appendix. As the number of observations at a given distance, a given hour of the day, and on a given frequency, are generally too limited as yet to permit of significant generalizations as to the distribution of percentages of commercial circuits, the curves have been derived from computed values of signal-to-noise ratios and curves showing the correlation between these signal-to-noise ratios and the results of percentages of commerical time as actually obtained in practice. The curves of Figs. 3 to 6 were used for the signal data and curves similar to those of Fig. 10 for the noise.

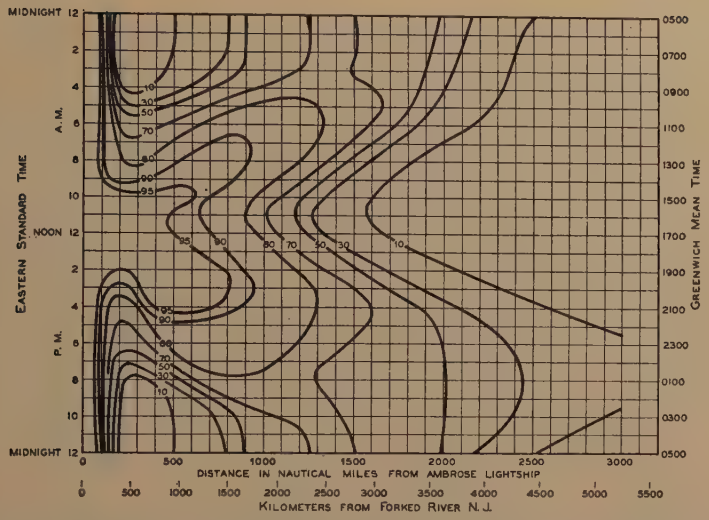

Fig. 12—Approximate variation of per cent commercial time with distance and time of day derived from idealized curves of signal field, data of diurnal variation of radio noise, and data on commercial circuits correlated with signal-to-noise ratios. Transmission from ships in north Atlantic shipping lanes to Forked River, New Jersey. Radiated power approximately 250 watts. Receiving antenna arrays used. Frequency 8.8 mc. Winter, 1930–1931.

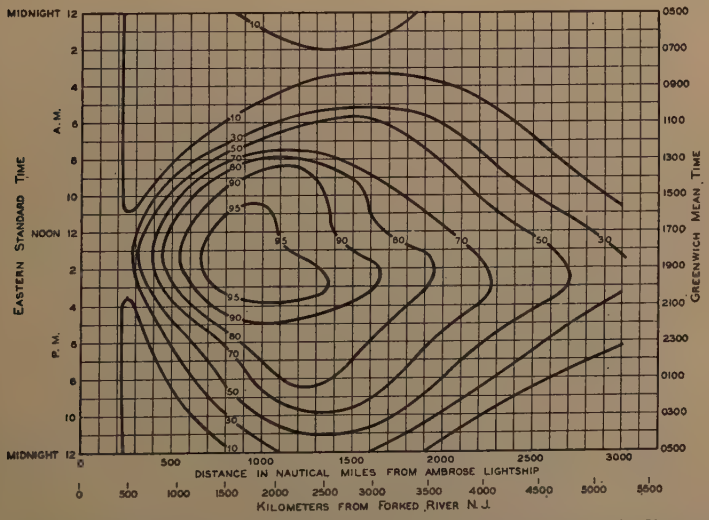

Fig. 13—Approximate variation of per cent commercial time with distance and time of day derived from idealized curves of signal field, data of diurnal variation of radio noise, and data on commercial circuits correlated with signal-to-noise ratios. Transmission from ships in north Atlantic shipping lanes to Forked River, New Jersey. Radiated power approximately 250 watts. Receiving antenna arrays used. Frequency 13.2 mc. Winter, 1930–1931.

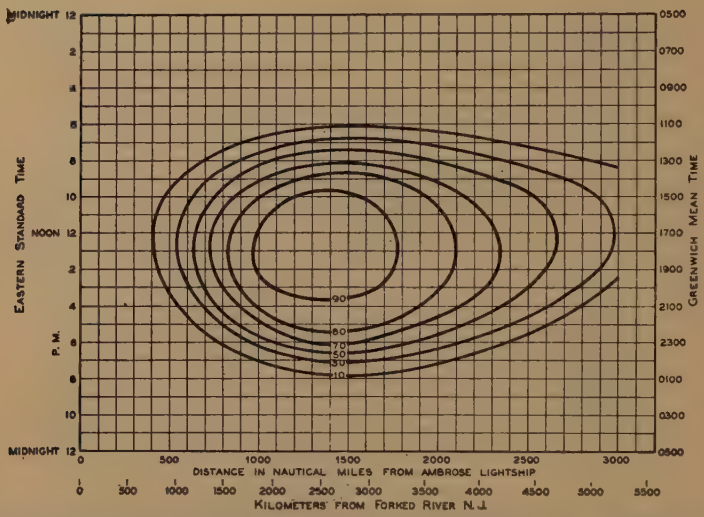

Fig. 14—Approximate variation of per cent commercial time with distance and time of day derived from idealized curves of signal field, data of diurnal variation of radio noise, and data on commercial circuits correlated with signal-to-noise ratios. Transmission from ships in north Atlantic shipping lanes to Forked River, New Jersey. Radiated power approximately 250 watts. Receiving antenna arrays used. Frequency 17.6 mc. Winter, 1930–1931.

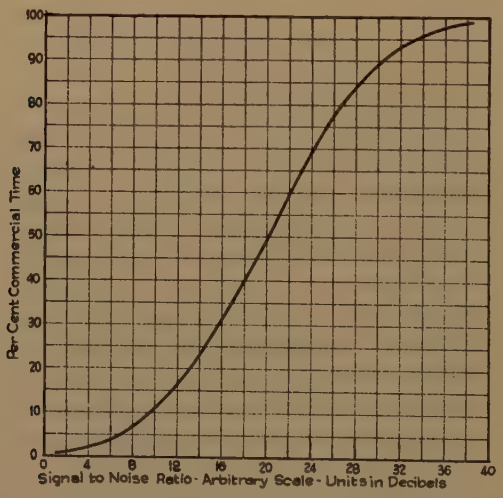

Fig. 15—Variation of per cent commercial time with signal-to-noise ratios.

The relation of the per cent of commercial time with the variation of signal-to-noise ratios is shown in Fig. 15. This permits us to evaluate approximately the effect of changes in radiated power upon the commercial results of the radio circuit. For example, with a reduction of 10 db in the signal-to-noise ratios, such as would result from substituting a 50-watt radio transmitter for a 500-watt transmitter, the percentage of commercial time of a circuit with, say, 80 per cent would be reduced to 30 per cent.

The distributions of commercial time apply, of course, only to the particular transmission path and with the particular terminal arrangements. To enable the results to be more generally applicable requires a further study in evaluating the antenna gains and other factors. Such studies are in progress.

Appendix

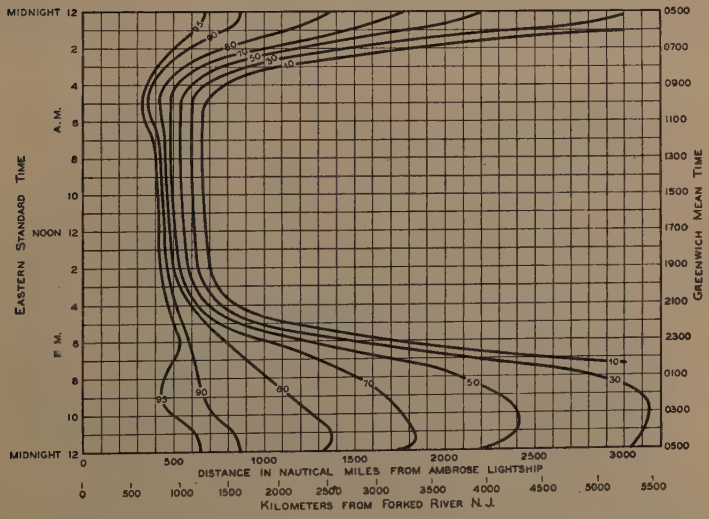

Fig. 16—Approximate variation of per cent commercial time with distance and time of day derived from idealized curves of signal field, data of diurnal variation of radio noise, and data on commercial circuits correlated with signal-to-noise ratios. Transmission from ships in north Atlantic shipping lanes to Forked River, New Jersey. Radiated power approximately 250 watts. Receiving antenna arrays used. Frequency 4.2 mc. Spring and fall, 1930–1931.

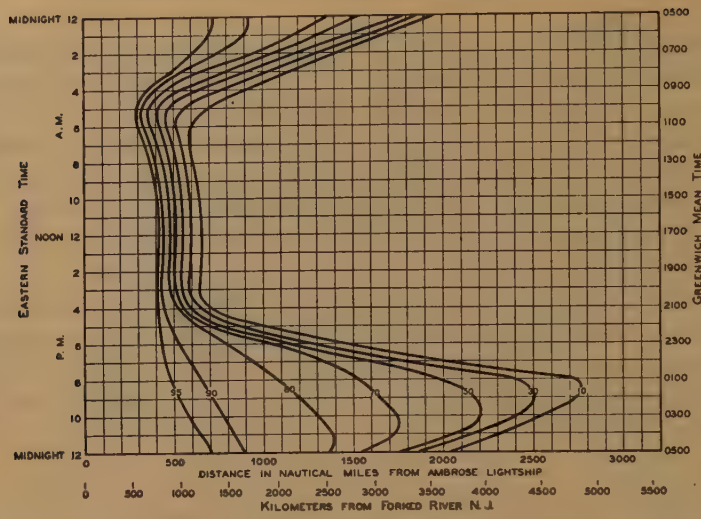

Fig. 17—Approximate variation of per cent commercial time with distance and time of day derived from idealized curves of signal field, data of diurnal variation of radio noise, and data on commercial circuits correlated with signal-to-noise ratios. Transmission from ships in north Atlantic shipping lanes to Forked River, New Jersey. Radiated power approximately 250 watts. Receiving antenna arrays used. Frequency 4.2 mc. Summer, 1930–1931.

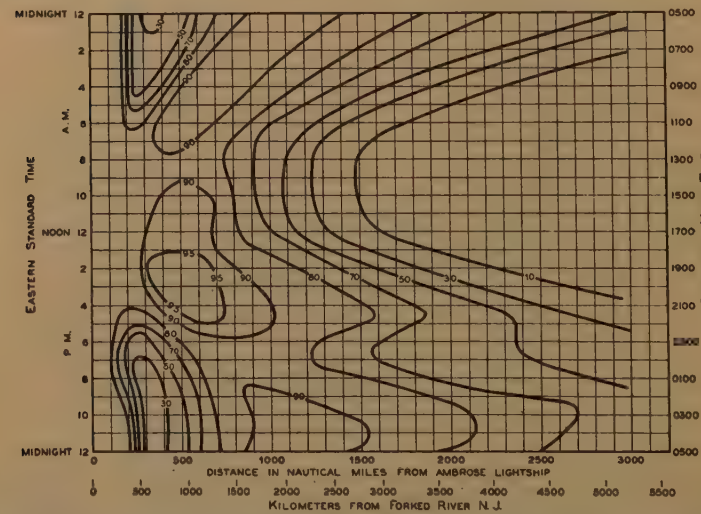

Fig. 18—Approximate variation of per cent commercial time with distance and time of day derived from idealized curves of signal field, data of diurnal variation of radio noise, and data on commercial circuits correlated with signal-to-noise ratios. Transmission from ships in north Atlantic shipping lanes to Forked River, New Jersey. Radiated power approximately 250 watts. Receiving antenna arrays used. Frequency 8.8 mc. Spring and fall, 1930–1931.

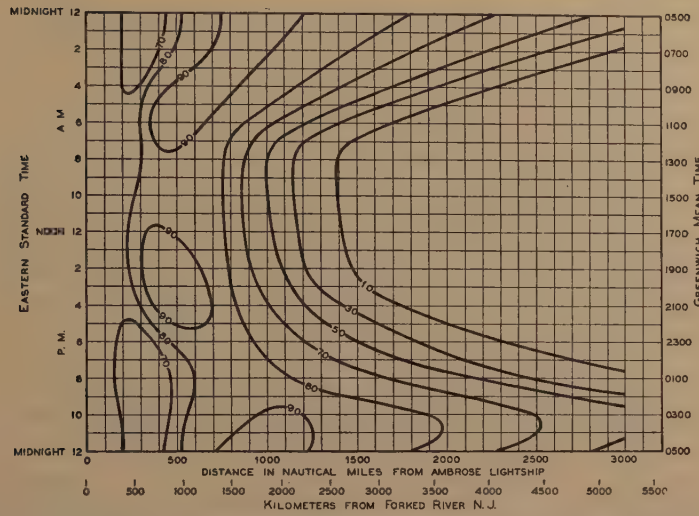

Fig. 19—Approximate variation of per cent commercial time with distance and time of day derived from idealized curves of signal field, data of diurnal variation of radio noise, and data on commercial circuits correlated with signal to noise ratios. Transmission from ships in north Atlantic shipping lanes to Forked River, New Jersey. Radiated power approximately 250 watts. Receiving antenna arrays used. Frequency 8.8 mc. Summer, 1930–1931.

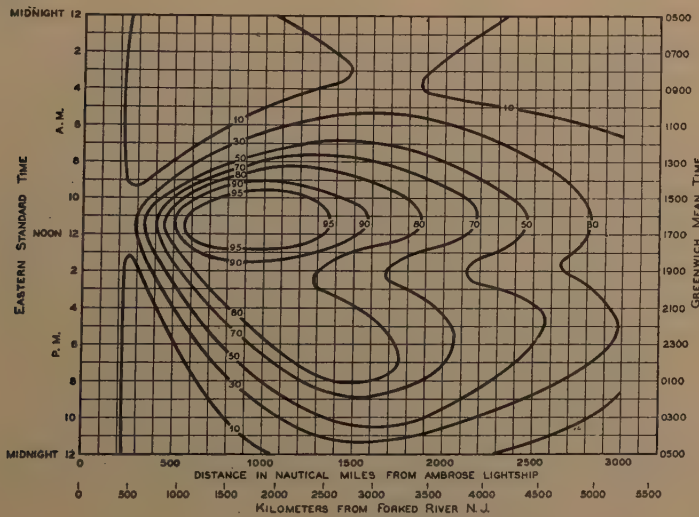

Fig. 20—Approximate variation of per cent commercial time with distance and time of day derived from idealized curves of signal field, data of diurnal variation of radio noise, and data on commercial circuits correlated with signal-to-noise ratios. Transmission from ships in north Atlantic shipping lanes to Forked River, New Jersey. Radiated power approximately 250 watts. Receiving antenna arrays used. Frequency 13.2 mc. Spring and fall, 1930–1931.

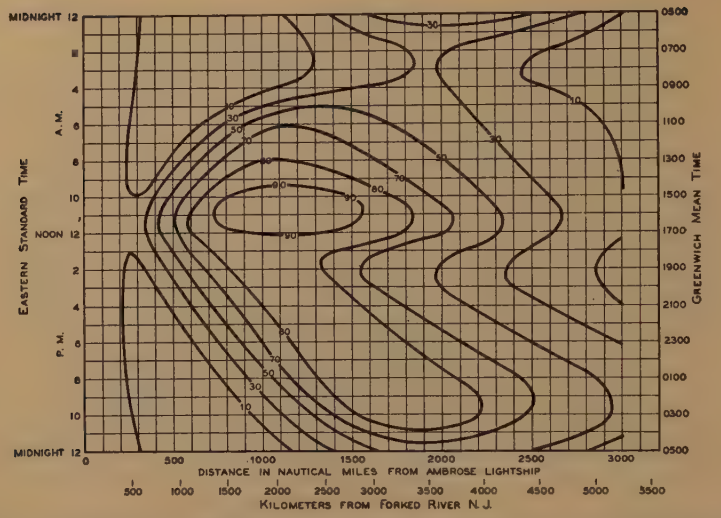

Fig. 21—Approximate variation of per cent commercial time with distance and time of day derived from idealized curves of signal field, data of diurnal variation of radio noise, and data on commercial circuits correlated with signal-to-noise ratios. Transmission from ships in north Atlantic shipping lanes to Forked River, New Jersey. Radiated power approximately 250 watts. Receiving antenna arrays used. Frequency 13.2 mc. Summer, 1930–1931.

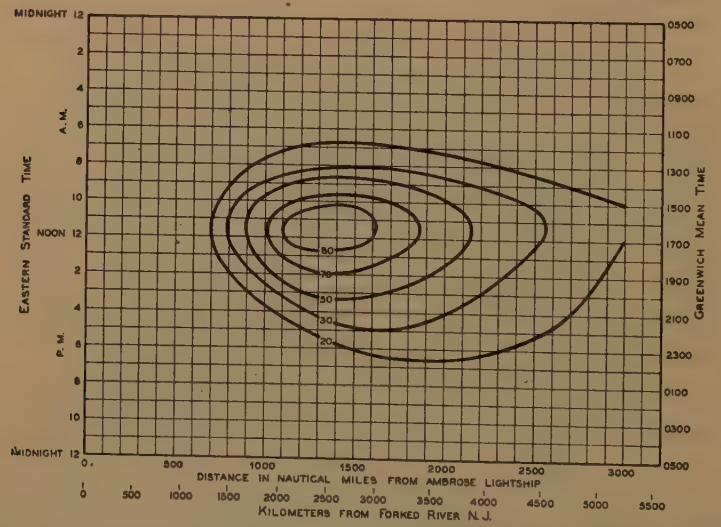

Fig. 22—Approximate variation of per cent commercial time with distance and time of day derived from idealized curves of signal field, data of diurnal variation of radio noise, and data on commercial circuits correlated with signal-to-noise ratios. Transmission from ships in north Atlantic shipping lanes to Forked River, New Jersey. Frequency 17.6 mc. Spring and fall, 1930–1931.

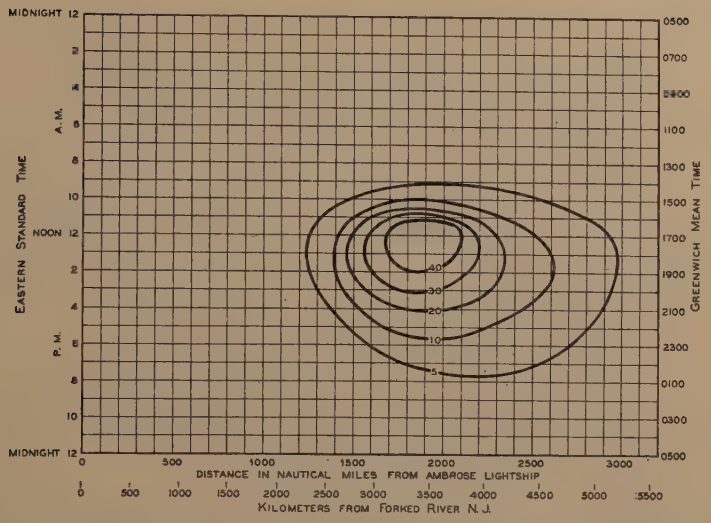

Fig. 23—Approximate variation of per cent commercial time with distance and time of day derived from idealized curves of signal field, data of diurnal variation of radio noise, and data on commercial circuits correlated with signal-to-noise ratios. Transmission from ships in north Atlantic shipping lanes to Forked River, New Jersey. Radiated power approximately 250 watts. Receiving antenna arrays used. Frequency 17.6 mc. Summer, 1930–1931.

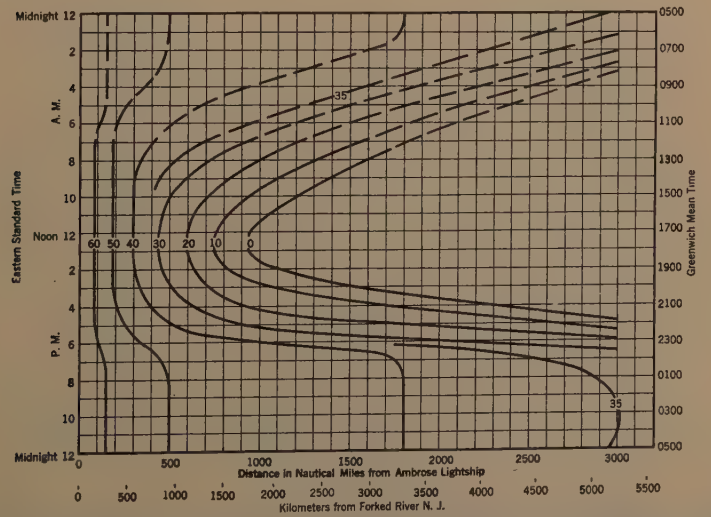
Midnight 12
Noon 12
A. M.
P. M.
Eastern Standard Time
Greenwich Mean Time
35
60 50 40 30 20 10 0
35
0 500 1000 1500 2000 2500 3000
Distance in Nautical Miles from Ambrose Lightship
0 500 1000 1500 2000 2500 3000 3500 4000 4500 5000 5500
Kilometers from Forked River N. J.

Fig. 24—Approximate variation of radio field strengths with distance and time of day. Contour lines indicate maximum expected field strengths in decibels above 1 microvolt per meter. Frequency 4.2 mc. Spring and fall, 1930–1931. Corrected to 1-kw radiated power. Solid lines indicate portions of contours determined by ship-shore data.

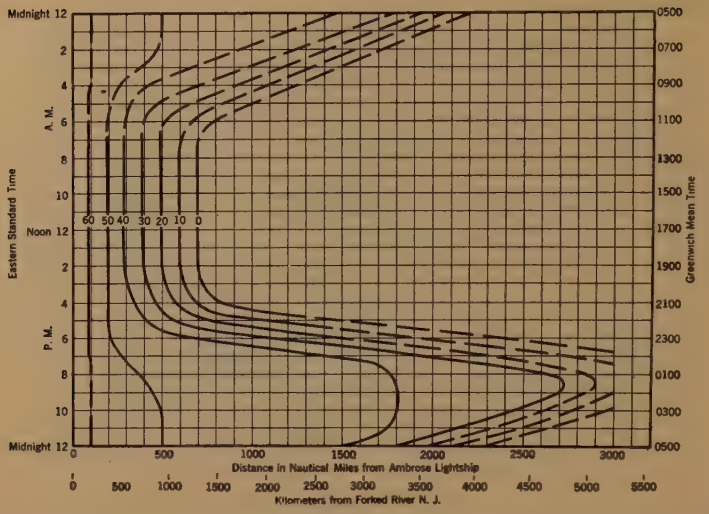

Fig. 25—Approximate variation of radio field strengths with distance and time of day. Contour lines indicate maximum expected field strengths in decibels above 1 microvolt per meter. Frequency 4.2 mc. Summer, 1930–1931. Corrected to 1-kw radiated power. Solid lines indicate portions of contours determined by ship-shore data.

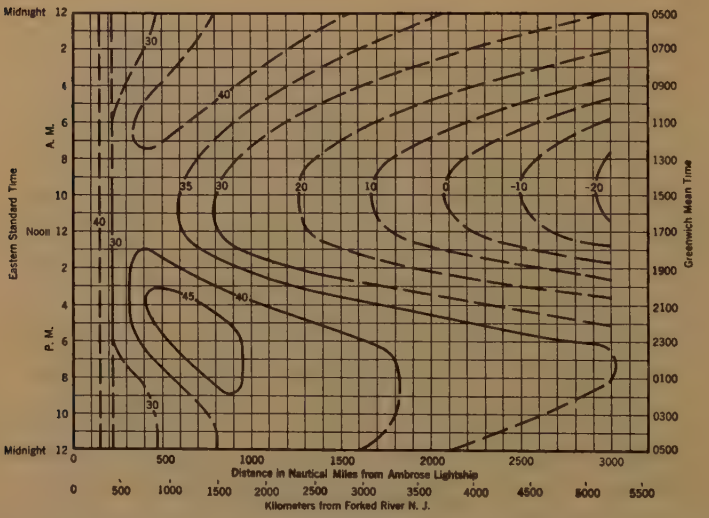

Fig. 26—Approximate variation of radio field strengths with distance and time of day. Contour lines indicate maximum expected field strengths in decibels above 1 microvolt per meter. Frequency 8.8 mc. Spring and fall, 1930–1931. Corrected to 1-kw radiated power. Solid lines indicate portions of contours determined by ship-shore data. Information on diurnal characteristic at 3000 miles supplemented by point-to-point transmission.

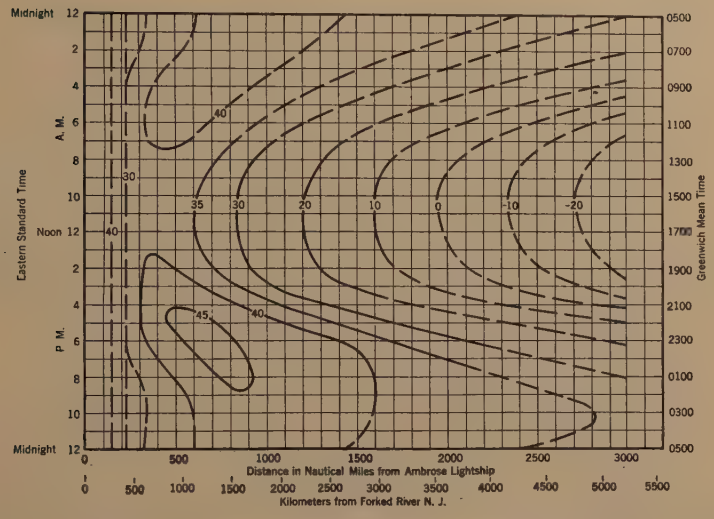

Fig. 27—Approximate variation of radio field strengths with distance and time of day. Contour lines indicate maximum expected field strengths in decibels above 1 microvolt per meter. Frequency 8.8 mc. Summer, 1930–1931. Corrected to 1-kw radiated power. Solid lines indicate portions of contours determined by ship-shore data. Information on diurnal characteristic at 3000 miles supplemented by point-to-point transmission.

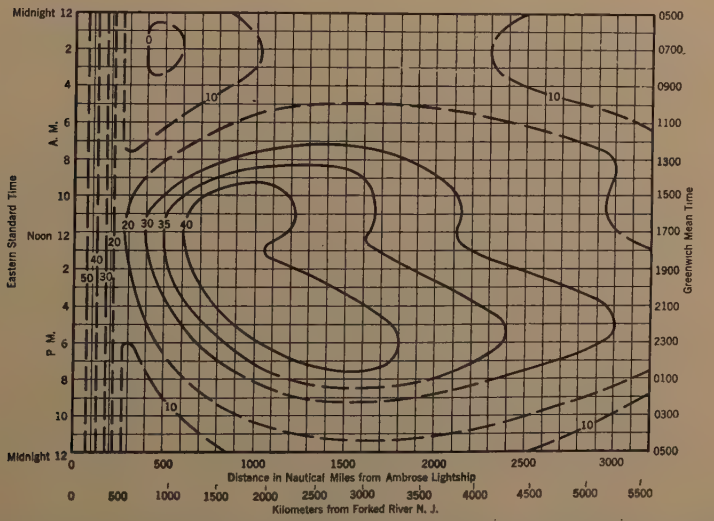

Fig. 28—Approximate variation of radio field strengths with distance and time of day. Contour lines indicate maximum expected field strengths in decibels above 1 microvolt per meter. Frequency 13.2 mc. Spring and fall, 1930–1931. Corrected to 1-kw radiated power. Solid lines indicate portions of contours determined by ship-shore data. Information on diurnal characteristic at 3000 miles supplemented by point-to-point transmission.

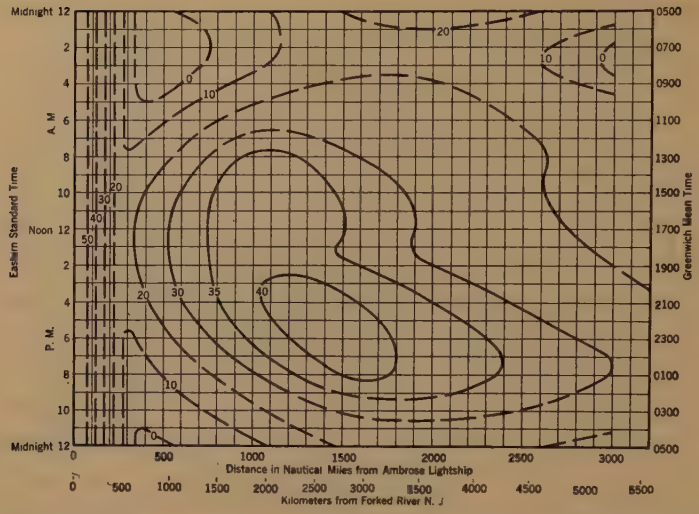

Fig. 29—Approximate variation of radio field strengths with distance and time of day. Contour lines indicate maximum expected field strengths in decibels above 1 microvolt per meter. Frequency 13.2 mc. Summer, 1930–1931. Corrected to 1-kw radiated power. Solid lines indicate portions of contours determined by ship-shore data. Information on diurnal characteristic at 3000 miles supplemented by point-to-point transmission.

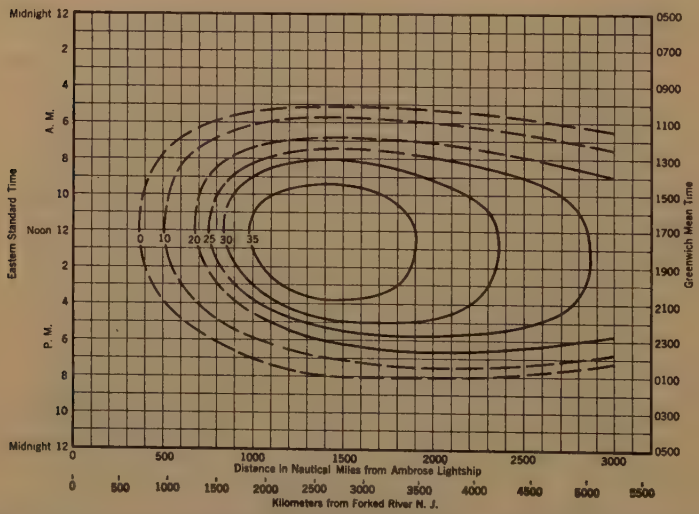

Fig. 30—Approximate variation of radio field strengths with distance and time of day. Contour lines indicate maximum expected field strengths in decibels above 1 microvolt per meter. Frequency 17.6 mc. Spring and fall, 1930–1931. Corrected to 1-kw radiated power. Solid lines indicate portions of contours determined by ship-shore data. Information on diurnal characteristic at 3000 miles supplemented by point-to-point transmission.

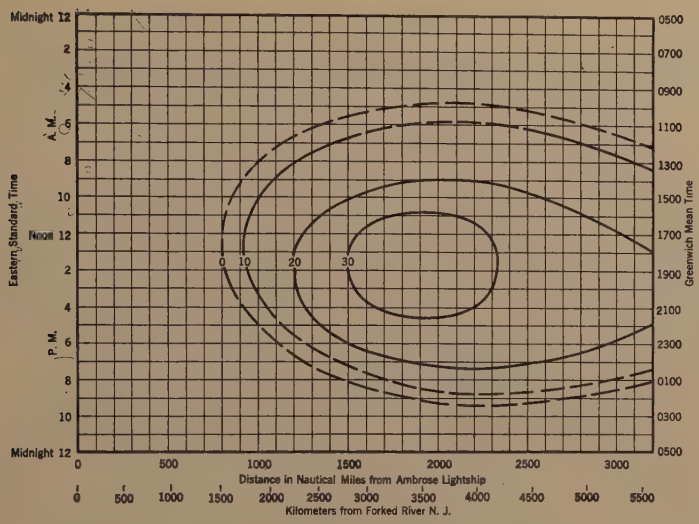

Fig. 31—Approximate variation of radio field strengths with distance and time of day. Contour lines indicate maximum expected field strengths in decibels above 1 microvolt per meter. Frequency 17.6 mc. Summer, 1930–1931. Corrected to 1-kw radiated power. Solid lines indicate portions of contours determined by ship-shore data. Information on diurnal characteristic at 3000 miles supplemented by point-to-point transmission.

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Volume 21, Number 1 *January, 1933*

SHORT-WAVE TRANSMISSION TO SOUTH AMERICA*

By

C. R. Burrows and E. J. Howard
(Bell Telephone Laboratories, New York City)

Summary *—The results of a year's survey of transmission conditions between New York and Buenos Aires in the short-wave radio spectrum are presented in this article. Surfaces showing the received field strength as a function of time of day and frequency are given. These show that frequencies between 19 and 23 megacycles were best for daytime transmission, and those between 8 and 10 megacycles for nighttime transmission. A transition frequency was required in the early morning, but the useful periods of the day and night frequencies overlapped in the evening.*

No variations that could definitely be traced to a seasonal effect were found. This path is much less affected by solar disturbances than the transatlantic.

Frequencies above 30 megacycles appear to have but little commercial value over this path. Frequencies a few megacycles higher could not be received.

Introduction

BEFORE the establishing of telephone service between the Bell System in this country and the network of the International Telephone and Telegraph Company in Argentina, Chile, and Uruguay, an investigation of short-wave transmission over this path was considered desirable. Although a considerable amount of data on short-wave transmission had been accumulated over a period of several years, mostly over the North Atlantic path between New York and London,[1] there were several ways in which this path differed from any that had at that time been investigated with quantitative receiving apparatus.

Some of the differences in the two paths are as follows:

1. The contemplated north-south circuit is one and one-half times times as long as the east-west.
2. The difference in time between terminals of the north-south path is one hour as against five hours on the east-west path; the path of transmission is nearly parallel to the "shadow line" instead of at right angles to it.
3. The seasons in Argentina are opposite to ours, while those in England are the same as ours.
4. Approximately two-thirds of the path to South America is over land while that to England is almost entirely over water.

* Decimal classification: R113. Original manuscript received by the Institute, June 28, 1932.

[1] *See* C. R. Burrows, "The propagation of short radio waves over the North Atlantic," Proc. I.R.E., vol. 19, pp. 1634–1659, September, (1931).

In order to determine the nature of the differences in transmission twenty-four hour tests were conducted once a week from October 21, 1928, to May 17, 1929. From then until the conclusion of the test, November 1, 1929, the test consisted of transmission every other week during the most important sixteen hours of the day. The general type and procedure of these tests were the same as those employed in the investigation of transmission conditions to England. They included field strength, noise, and intelligibility measurements on 6.755, 10.55, 16.27, 21.42, 27.51 megacycles (44.4, 28.44, 18.44, 14, 10.9 meters).

The transmitter used was the same one employed in the trans-atlantic tests.[2] A power of approximately 5 kilowatts was radiated from vertical half-wave antennas for all frequencies except the two highest for which a power of only 1 kilowatt was radiated.

Field strength measurements were made on a set developed by H. T. Friis and E. Bruce and described in the PROCEEDINGS.[3] Antennas used with these sets consisted in each case of single vertical wire elements which were calibrated at each frequency against a standard loop as suggested in the above-mentioned article.

FIELD STRENGTH SURFACES

Average transmission conditions over this path are shown at a glance in Fig. 1. It gives the field strength as a function of the time of day and the frequency. Points within the two black[4] regions specify conditions when the signal is not received. The first represents the poor nighttime transmission on the higher frequencies; the second depicts the period in the daytime when the lower frequencies fail. The individual diurnal variation curves show a tendency for a daytime valley to be present even on 27 megacycles. Another fact that is averaged out in the surface is the tendency of the field on the lower frequencies to be slightly higher just after it comes in and before it goes out than it is during the rest of the useful period.

The change of seasons does not produce a marked effect in the transmission conditions over this path. This may be seen by comparing Figs. 2 and 3 which represent quarterly periods about a solstice and an equinox, respectively.[5] This behavior is due to the opposition of the

[2] *Loc. cit.*, page 1635.

[3] "A radio field strength measuring system for frequencies up to forty megacycles," PROC. I.R.E., vol. 14, pp. 507–521; August, (1926).

[4] The level of this region is probably much lower than the −20 decibels indicated at the boundary. The sensitivity of the measuring set and noise precluded the determination of the exact level.

[5] During the remainder of the year the test periods were reduced to bi-weekly, and later the unimportant commercial hours were omitted. The data do not indicate any difference between the quarters for the equinoxes or between the quarters for the two solstices.

seasons over the north and south halves of the path. Some seasonal effect probably remains in spite of this tendency toward cancellation, but it is of minor importance and difficult to separate from the large day-to-day variations resulting from other causes. It seems likely to the writers that for this path the solar cycle has a more marked effect

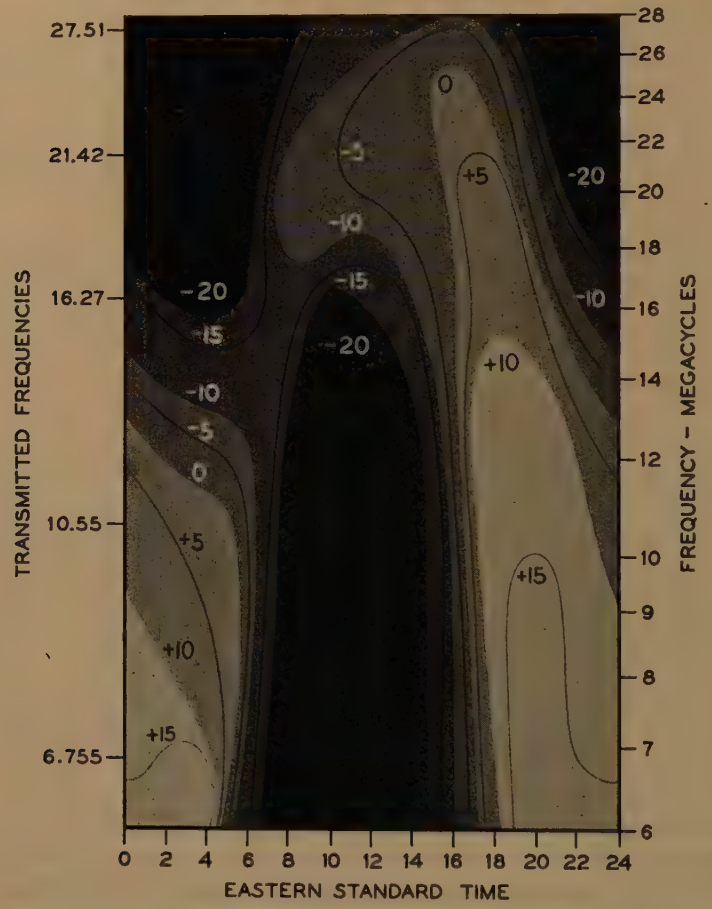

Fig. 1—Average field strength surface for October 12, 1928, to November, 1, 1929. Decibels above $1\mu v/m$ for 1 kilowatt radiated.

than the cycle of the seasons.[6] This is in marked contrast to conditions over paths well away from the equator, for example, those over the North Atlantic.[7]

[6] Conclusions should not be drawn from data from one year only in view of possible differences in solar activity, e.g., a 15-month solar cycle. *See* H. T. Stetson, "The influence of sunspots on radio reception," *Jour. Frank. Inst.*, vol. 210, pp. 403–419; October, (1930).

[7] Compare with Figs. 13 and 14 of reference 1.

All of the surfaces show that generally speaking frequencies between 19 and 23 megacycles were best during the daytime, and those between 8 and 10 megacycles were to be preferred at night. Sometimes lower frequencies gave higher field strengths but this advantage was usually offset by a higher noise level.

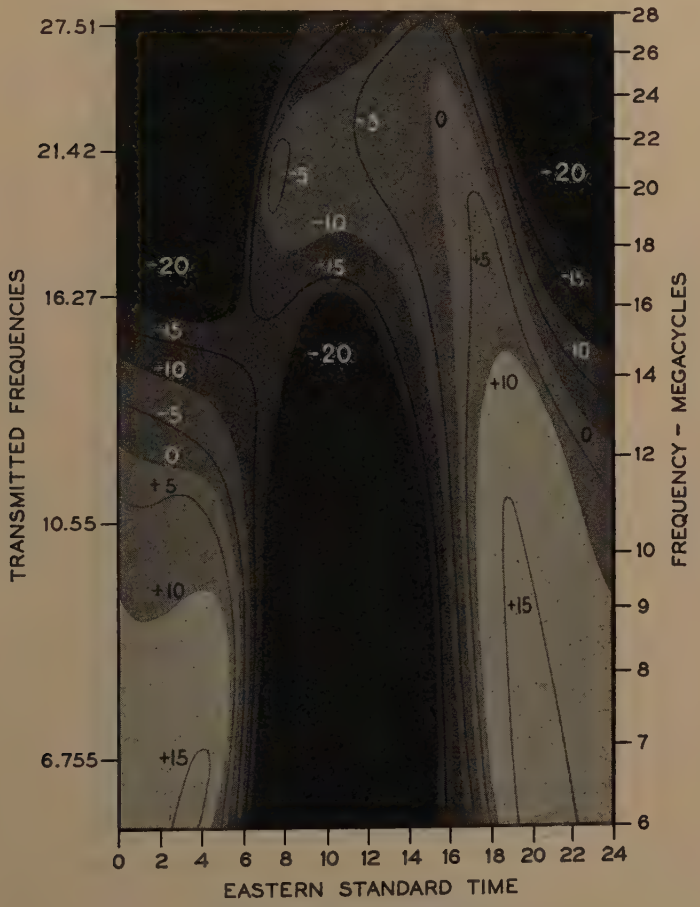

Fig. 2—Average field strength surface for solstice quarter November, 1928, to January, 1929. Decibels above $1\mu v/m$ for 1 kilowatt radiated.

When the daytime absorption on the lower frequencies is still pronounced on 21 megacycles, a higher frequency may give stronger fields. This was the case with the 27-megacycle frequency on several of the test days.

Another weak period occurs at the time when it is necessary to change from a night frequency to a day frequency (6 or 7 o'clock,

E.S.T.). This was the most difficult time for transmission during the year of these tests. There was usually a lapse of about two hours between the time of high fields on the night frequency and the time of high fields on the day frequency. The most help, at these times would come from an intermediate frequency such as 16 megacycles.

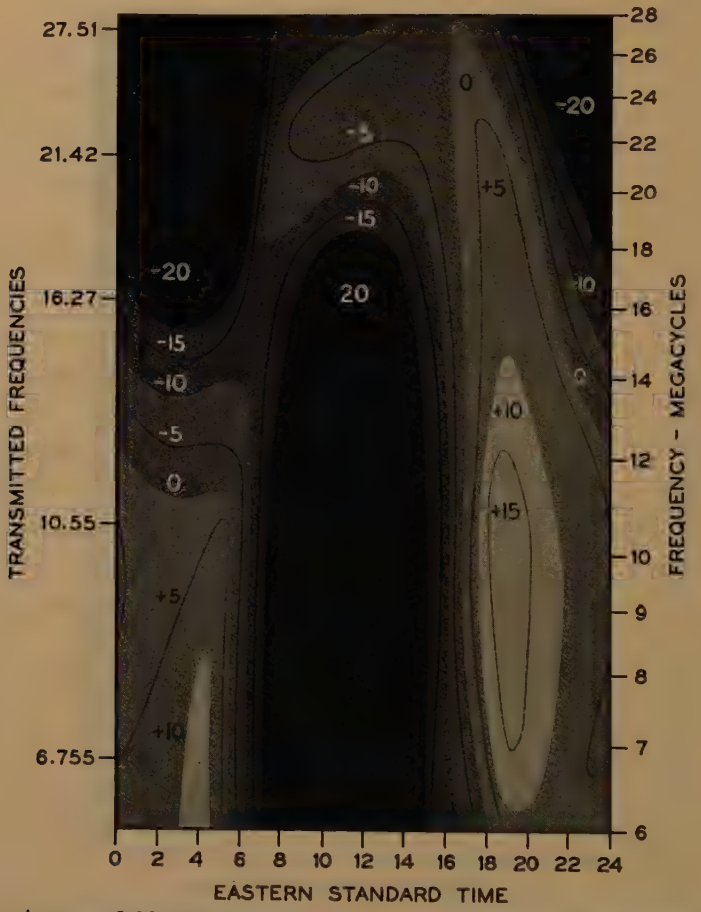

Fig. 3—Average field strength surface for equinox quarter February to April, 1929. Decibels above $1\mu v/m$ for 1 kilowatt radiated.

Day-to-day variations were very much less on the lower frequencies than on the higher ones. On the two lower frequencies, 6.7 and 10.5 megacycles, the times when the signal came in were very nearly the same on all of the test days. The same can be said of the time when it disappeared. Between these times the field strength curve was comparatively flat and high. On the two higher frequencies, 27 and 21

megacycles, on the other hand, the field strength curves showed considerable variation among themselves. Sometimes the curve on these frequencies had one high level region, sometimes it exhibited two or more peaks, and at other times the first peak would be absent resulting in weak signals during the early part of the period.

Fig. 1 indicates that between 14 and 18 megacycles a frequency can be chosen which is influenced both by the nighttime skip depression and by the midday absorption minimum. The combining of these opposing tendencies in one diurnal characteristic will result, so to speak, in the cancellation of the component having a twenty-four hour period, and will leave instead a second harmonic of the daily cycle. The test frequency 16.27 megacycles often exhibited this characteristic. Day-to-day variations in the transmitting medium changed the type of the characteristic on this frequency in an erratic manner. Sometimes it resembled a night frequency, though more often it had day frequency characteristics. It seems likely that a study of these fluctuations may assist in the determination of the causes of the day-to-day fluctuations in the medium.

A few tests were conducted on 31 and 36 megacycles. The 31-megacycle frequency was received on all of the test days at some time during the period 1000–1500 E.S.T. Field strengths as high as the maxima measured on 27 megacycles were sometimes obtained. The diurnal variation curves are so different for the different days that an average curve would be misleading. This frequency of 31 megacycles has little commercial value because of the irregularity of the time it is received.

Thirty-six megacycles was never received. The maximum signal at 36 megacycles during any of the tests must have been at least 15 decibels below that on 31 megacycles since a signal of this intensity could have been detected with the receiver employed. This would seem to indicate that the upper limiting frequency for this path lies near the range of 31 to 36 megacycles.

It may be well to point out that measurements of the type here presented are not independent of the type of transmitting and receiving antennas employed. This is due to the fact that the transmitting medium shows a preference for signals having a certain angle of departure and a certain angle of arrival. In order to determine the loss occasioned by the medium, the efficiency of the antenna in these directions must be taken into account. At best a complicated process, this becomes very difficult if not impossible when transmission occurs over several paths simultaneously as, in all probability, it usually does.

It is conceivable that these considerations may explain to some extent the erratic behavior of the highest frequencies used in these tests.

For 31 megacycles the preferred angles may be so near to the horizontal that the low efficiency of the antennas in this direction made reception very difficult. Potter and Friis[8] have found that 21-megacycle signals from South America are strongest for antennas with low angle polar characteristics in the vertical plane.

Disturbed Days

Transmission over this path has not been as adversely affected during periods of solar acitivity as that over the North Atlantic. The rela-

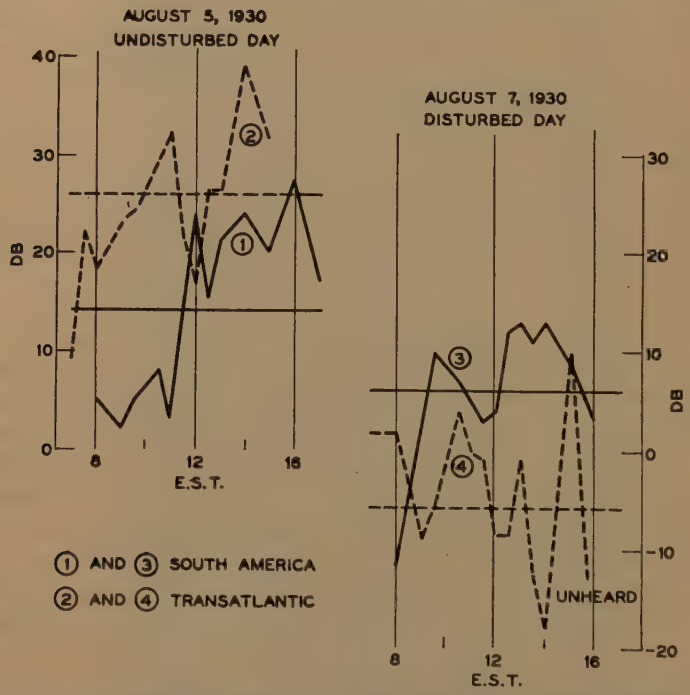

Fig. 4—Diurnal variation curves on the day frequency showing relative effects of solar activity on transatlantic and South American transmission. The reduction in field strengths on the disturbed day, August 7, 1930, is less pronounced on the South American path (compare curves (1) and (3)) than on the transatlantic path (compare curves (2) and (4)).

tive effects of solar activity on the two circuits are illustrated in Fig. 4. These curves show the received field strength on the day frequency for the two paths. The curves for August 7, 1930, show the conditions during a period of solar activity. The curves for August 5, 1930, show conditions on an undisturbed day preceding this period of solar activity.

[8] "Some effects of topography and ground on short-wave reception," Proc. I.R.E., vol. 20, pp. 699–721; April, (1932).

These curves show that transmission conditions were much more adversely affected on the transatlantic path than on the South American one. Although this was one of the more severe solar disturbances, telephone communication to South America was not seriously affected.

Transmission Conditions of Individual Days

It may be interesting to supplement the average conditions set forth in Figs. 1 to 3 with the typical daily curves shown in Figs. 5 and 6. An inspection of these curves show that marked differences occur from day to day.

Without attempting to illustrate each point, the following characteristics may be mentioned in order to portray different types of the variations which occur:

(1) The daytime absorption minimum which is always present for the lower frequencies frequently seems to extend as high as 21 megacycles and perhaps even higher. At these higher frequencies the depth of this minimum is not ordinarily sufficient to interfere with commercial traffic.

(2) At 27 megacycles, great variability exists in the duration of the period when strong signals are received. This is well illustrated by Figs. 5 and 6.

(3) 16 megacycles occasionally can be received through the full 24 hours (Fig. 5). Usually it takes on the characteristics of a day or night frequency, and sometimes it shares the weak periods of both day and night waves.

(4) The low frequencies (6.755 and 10.55, inclusive) behave the most consistently as regards the times of the beginning and ending of the useful period.

Comparison with Transatlantic Transmission

With the aid of the data presented in a previous article[8] the following comparisons may be made between transmission over the North Atlantic and to South America:

(1) With similar facilitaties, it should be possible to maintain a better grade of service throughout the twenty-four hours between New York and Buenos Aires than between New York and London.

(2) On the New York-Buenos Aires circuit, the interruptions due to disturbed solar conditions were fewer and of shorter duration than on the circuit to London.

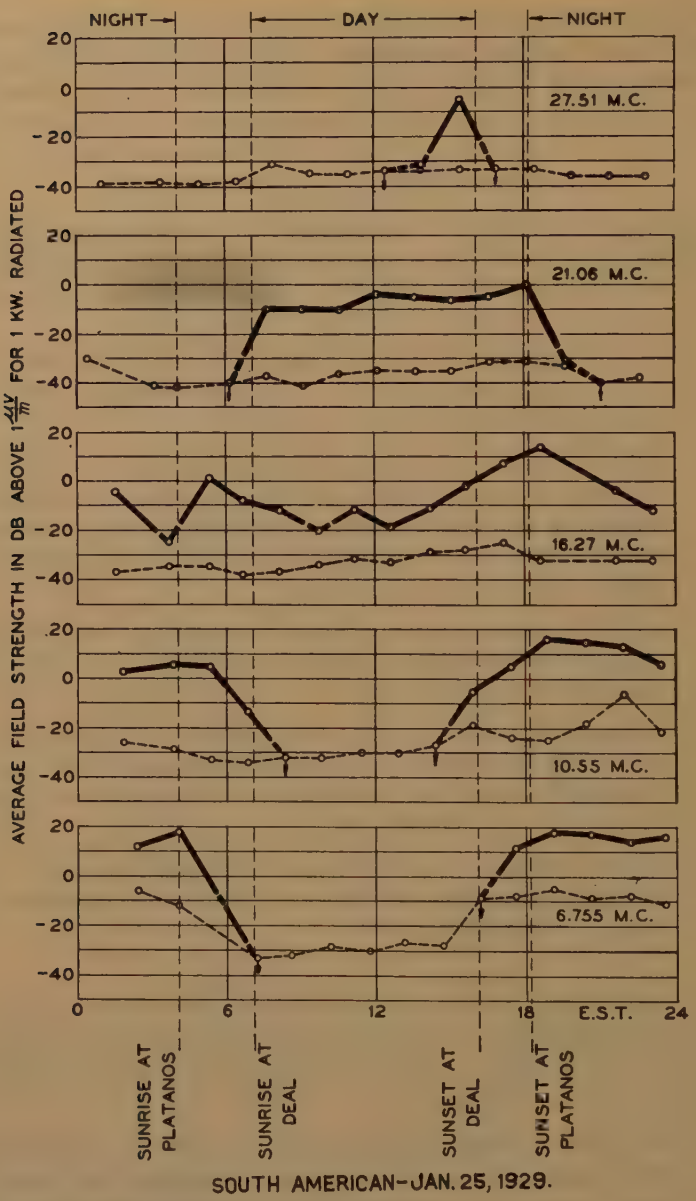

Fig. 5—January 25, 1929.

27 megacycles. One narrow peak at 1500.
21 megacycles. Flat plateau 0700–1900.
16 megacycles. Received all day, shows both absorption and skip depressions.
10 megacycles. Rounded plateau 1500–0700 with depression.
6 megacycles. Rounded plateau 1600–0600 with depression.

Dotted curves indicate minimum measurable signal.

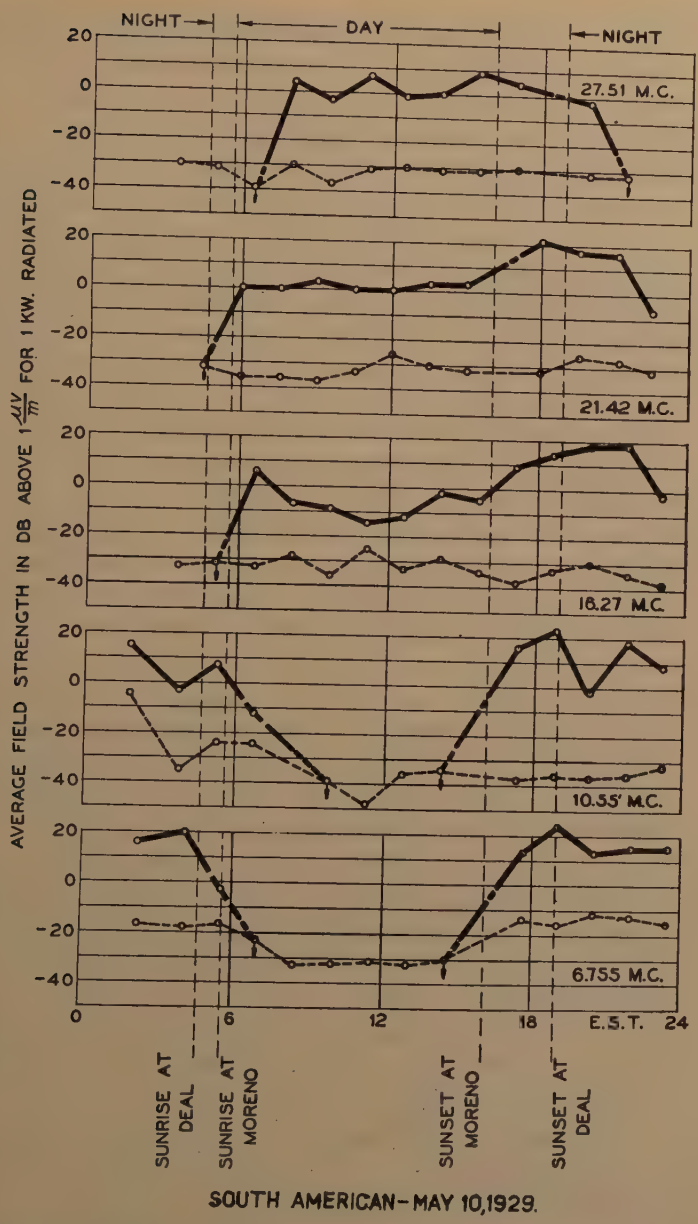

Fig. 6—May 10, 1929.

27 megacycles. Plateau 0700–2100 with three minor peaks.
21 megacycles. Rising plateau 0500–2300.
16 megacycles. Plateau 0600–2400 with wide depression around noon, rising slope after first peak.
10 megacycles. Plateau 1500–0700 with three depressions.
6 megacycles. Plateau 1600–0600 with slight depression.
Dotted curves indicate minimum measurable signal.

(3) There was no weak period at sunset in the New York-Buenos Aires transmission as there is in the New York-London transmission.

(4) For frequencies around 27 megacycles there is better transmission to Buenos Aires than to London. This is due in part to the greater length of transmission path,[9] but may also be due to other factors, such as more intense illumination from the sun and greater separation from the magnetic poles.

(5) At midday there is greater absorption on the lower and intermediate frequencies on the South American than on the transatlantic path. This absorption dip was even noticeable on all the transmitted frequencies 6.7, 10, 16, 21, and 27 megacycles on more than half of the test days. The greater length of path and more intense illumination from the sun are in the right direction to cause this greater absorption.

(6) Transmission conditions to Buenos Aires on a given frequency are in many ways comparable with those to London on a frequency 33 per cent lower. This was particularly noticeable at the transition frequencies (about 16 megacycles to Buenos Aires and about 10 megacycles to London).

(7) For the South American circuit the changes between night and day transmission conditions are more abrupt than on the transatlantic path. This is presumably due to the fact that the times of sunrise and sunset are more nearly the same at the transmitter and receiver on the former than on the latter.

(8) The seasonal change in transmission conditions is very much less pronounced between New York and Buenos Aires than between New York and London. In fact no variations that could unmistakably be attributed to seasonal changes have been found for the former path.

(9) The results of these tests do not substantiate the view that transmission over this path is more difficult during an equinox than during a solstice.

(10) The atmospheric noise has the following general characteristics at both locations:

(a) the noise increases with decrease in frequency,

[9] The reason for this is obtained from consideration of the fact that, for the same conditions of the Kennelly-Heaviside layer, the higher frequencies are not returned to the earth within as short a distance as the lower frequencies. That is, the skip distance is longer for the higher frequencies. Greater ionization due to more intense illumination from the sun tends to decrease the skip distance.

(b) the range of its diurnal variation increases with decrease in frequency,

(c) the maxima of the noise and signal diurnal variation occur at approximately the same time, and likewise their minima.

Acknowledgment

The authors wish to acknowledge the coöperation of A. Decino who was responsible for the calculations, J. W. Sanborn of the International Telephone and Telegraph Company, who assisted at the receiver, and of A. E. Kerwien who was in charge at the transmitter.

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TRANSMISSION CURVES OF HIGH-FREQUENCY NETWORKS*

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Summary *—The aim of this article is to deduce the design data on frequency transmission, which are necessary in designing the networks for high-frequency modulated waves.*

In the first section the general laws of current variations in circuits tuned to carrier frequency with and without tube generator are derived; the interdependence of current curves of various circuits forming a given network is also clarified.

The second section gives the derivation of the transmission curve equation for a two-circuit system, analyzes the curve shape as dependent upon the parameters of the circuits and the coupling factor between them, and also explains the part the tube generator plays in restricting the field of application of formulas derived. Moreover, the equation of the current curve for inaccurately tuned circuits is given, which shows that inaccurate tuning results in asymmetrical curves and for that reason is not to be recommended.

The third section contains the derivation of the transmission curve equation for a three-circuit system and the analysis of the curve shape as dependent on parameters of circuits and coupling factors between them.

Introduction

THE extensive use of high-frequency modulated waves in broadcasting, television, and telegraphy makes the problem of undistorted frequency transmission very important in the whole range of waves, from the longest to the ultra-short. The literature on the subject, however, does not give sufficient data convenient for an engineering design. The present article tries partially to fill up this gap. All the principal theoretical statements (derived by the author in 1929) have been checked by experiments with transmitting equipment. The experimental study has shown the possible divergencies between practice and theory, the causes of these divergencies and the field within which theoretical statements coincide with experimental data.

Moreover, a number of conclusions useful for designing receiving apparatus is incorporated in this article.

I. General Relations

First, consider a general system of N circuits, each circuit (number n) being coupled to as many as two adjacent, the preceding (number $n-1$) and succeeding (number $n+1$), circuits. The first circuit is con-

* Decimal classification: R140. Original manuscript received by the Institute, July 11, 1932.

nected to a tube generator (Fig. 1), instead of which, as tube-generator theory proves, a source of e.m.f. with suitable internal resistance R_i/α can be used; herein R_i is the internal plate resistance of the tube as

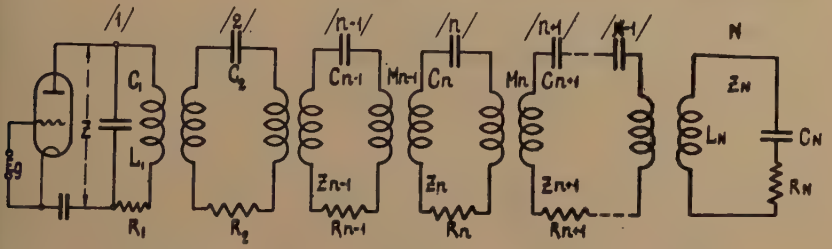

Fig. 1

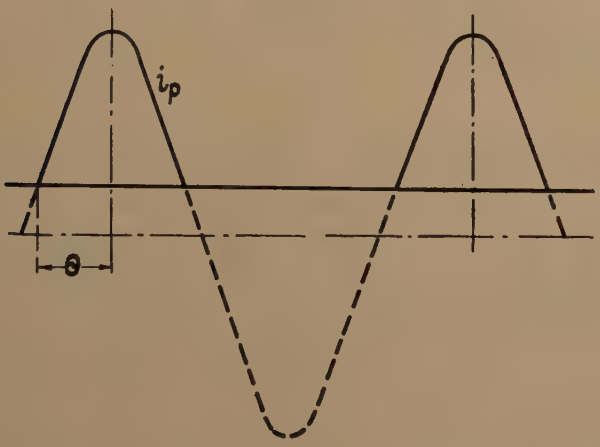

Fig. 2

determined by its static curves, and α is the plate-current fundamental wave factor of either the B or C class of amplification. (Fig. 2.)

$$\alpha = \frac{2\theta - \sin 2\theta}{2\pi}$$

(When class B amplification is used $\theta = \pi/2$; $\alpha = 0.5$.)

In case of reception, networks are possible in which an e.m.f. is directly applied to the first circuit. Thus, the two types of networks are obtained. (Figs. 3 and 4.)

For the sake of simplicity, inductive coupling between various circuits is shown. It will be shown later, that capacitive or capacitive-inductive coupling gives practically the same results as inductive, as far as the frequency transmission curve is concerned. Frequency trans-

mission through a given network can be judged by the resonance curve, which represents values of current in the last circuit (e.g. antenna current) plotted against the driving frequency, the e.m.f. amplitude being kept constant.

Each circuit with elements independent of frequency is supposed to be separately tuned to the carrier frequency. It will be shown later that such tuning is necessary to avoid distortion due to phase modulation.

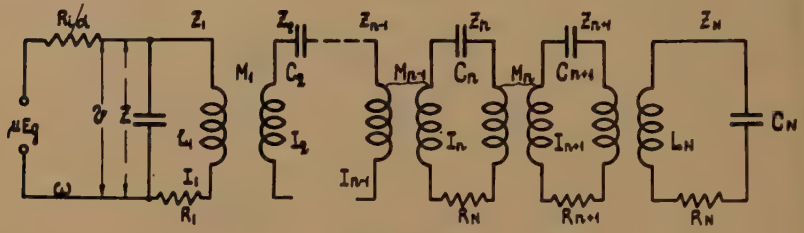

Fig. 3

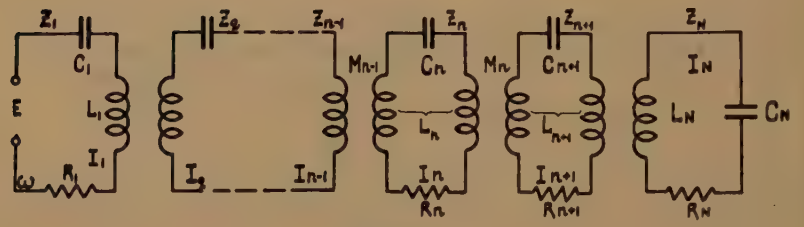

Fig. 4

The following notation will be used:

$\omega_0 = 2\pi f_0$ — resonance frequency (carrier)

$\omega = \omega_0 + \Omega$ — pulsation (of side bands) differing from resonance frequency by an audio frequency $\Omega = 2\pi F$ (here Ω has both signs $\pm$)

$\delta_n = R_n\omega_0 C_n = \dfrac{R_n}{\omega_0 L_n}$ — decrement coefficient of circuit number n

$K_n = \dfrac{M_n}{\sqrt{L_n L_{n+1}}}$ — coupling factor between circuit number n and circuit number $n+1$

$\delta_n' = \delta_n + \dfrac{K^2}{\delta'_{n+1}}$ — total decrement coefficient of the number n circuit, in which K^2/δ'_{n+1} is the reduced decrement coefficient of the next circuit

Z_0 equivalent impedance of tube generator load under resonance conditions (this value is predetermined by the power required, the characteristics and operating conditions of the generator, and not by network configuration)

Z same at frequency $\omega = \omega_0 + \Omega$

$Z_n = R_n + j\left(\omega L_n - \frac{1}{\omega C_n}\right)$ circuit number n impedance

$Z_{n0} = R_n$ same at resonance conditions

$R_n' = R_n + \frac{\omega_0^2 M_n^2}{R'_{n+1}}$ total resistance of the number n circuit under resonance conditions

$Z_n' = Z_n + \frac{\omega^2 M_n^2}{Z'_{n+1}}$ total impedance of the number n circuit.

At the resonance frequency ω_0 the latter impedance will be equal to the total resistance, viz:

$$Z_{n0}' = R_n'.$$

Neglecting Ω^2/ω_0^2 as compared to 1 we have:

$$Z_n = R_n + j\frac{(\omega_0 + \Omega)^2 L_n C_n - 1}{\omega C_n} \cong R_n + j\frac{2\Omega}{\omega}\omega_0 L_n$$

$$Z_n = R_n\left(1 + j\frac{2\Omega}{\omega\delta_n}\right). \tag{1}$$

In case of single-circuit tube generator, the equivalent of the load under resonance conditions is:

$$Z_0 = \frac{R_1 + j\omega_0 L_1}{j\omega_0 C_1 R_1} = \frac{L_1}{C_1 R_1} - j\frac{1}{\omega_0 C_1}.$$

Neglecting the reactive component which is considerably smaller than the other, we have:

$$Z_0 \cong \frac{L_1}{C_1 R_1}. \tag{2}$$

In practice the circuit is tuned to minimum direct-current component of the plate current, which usually corresponds to

$$Z_0 = \frac{L_1}{C_1 R_1}$$

at the frequency

$$\omega_0'^2 = \frac{1}{L_1C_1} - \frac{R_1^2}{L_1^2}.$$

As these frequencies diverge from each other but little, the assumption that the circuit is tuned to frequency

$$\omega_0^2 = \frac{1}{L_1C_1}$$

will not introduce any substantial error.

Making the same assumptions, the impedance Z_0 at frequency ω_0 is given by

$$Z = \frac{L_1}{C_1Z_1} \tag{3}$$

in which,

$$Z_1 = R_1\left(1 + j\frac{2\Omega}{\omega\delta_1}\right).$$

Therefore,

$$\frac{Z_0}{Z} \cong \frac{Z_1}{R_1} = 1 + j\frac{2\Omega}{\omega\delta_1}. \tag{4}$$

For the Nth circuit of the system (Fig. 3)

$$Z = \frac{Z_cZ_M}{Z_1'}$$

in which,

$$Z_c = 1/j\omega C_1,\ Z_M' = R_1 + j\omega L_1 + \frac{\omega^2M_1^2}{Z_1'}$$

$$\therefore \quad \frac{Z_0}{Z} = \frac{\omega}{\omega_0}\,\frac{Z_{M0}}{Z_M}\,\frac{Z_1'}{R_1'}. \tag{5}$$

The current in the primary circuit is

$$I = \mu E_g \frac{1}{\frac{R_i}{\alpha} + Z}\,\frac{Z_c}{Z_1'}.$$

Designating the current at resonance frequency by I_{10}, we will have:

$$\frac{I_{10}}{I_1} = \frac{Z + R_i/\alpha}{Z_0 + R_i/\alpha}\,\frac{Z_1'}{R_1'}\,\frac{\omega}{\omega_0}. \tag{6}$$

It should be noted that the assumption $\alpha = \text{const.}$ for the whole range of frequencies cannot introduce any considerable error, because the plate-current curve of generator tubes is primarily determined by the amplitude of grid voltage, and not by the plate reaction unless plate overvoltage exists, which case will be considered farther. The relation (5) can be simplified as we can without appreciable error write:

$$Z_M' \cong j\omega L_1.$$

(A similar simplification was made in deriving Z_0.) Then,

$$Z = Z_0 \frac{R_1'}{Z_1'} \cdot \tag{7}$$

Substituting this in (6) and to simplify, writing

$$\beta = \alpha Z_0 / R_i. \tag{8}$$

We obtain

$$\frac{I_{10}}{I_1} = \frac{\omega}{\omega_0} \frac{1}{1+\beta}\left(\frac{Z_1'}{R_1'} + \beta\right). \tag{9}$$

The current in the circuit number n:

$$I_n = \frac{I_{n-1}\omega M_{n-1}}{Z_n'} \cdot$$

It is easy to see the following relation between the currents in the last and first circuits (Fig. 3):

$$I_N = I_1 \frac{\omega^{N-1} M_1 M_2 \cdots M_{N-1}}{Z_2' Z_3' \cdots Z_N} \cdot$$

Let I_{N0} current at resonance frequency, then

$$\frac{I_{N0}}{I_N} = \frac{I_{10}}{I_1}\left(\frac{\omega_0}{\omega}\right)^{N-1} \frac{Z_2' Z_3' \cdots Z_N}{R_2' R_3' \cdots R_N} \cdot$$

Thus by (9) we obtain

$$\frac{I_{N0}}{I_N} = \left(\frac{\omega_0}{\omega}\right)^{N-2} \frac{1}{1+\beta}\left[\frac{Z_1' Z_2' \cdots Z_N}{R_1' R_2' \cdots R_N} + \beta \frac{Z_2' Z_3' \cdots Z_N}{R_2' R_3' \cdots R_N}\right]. \tag{10}$$

It is evident that the exponent of (ω_0/ω) depends on the number of inductive or capacitive couplings.

The case of e.m.f. directly impressed upon the primary circuit (Fig. 4) may be considered as the particular case in which $\beta = 0$. Then:

$$\frac{I_{10}}{I_1} = \frac{Z_1'}{R_1'}$$

and therefore,

$$\frac{I_{N0}}{I_N} = \left(\frac{\omega_0}{\omega}\right)^{N-1} \frac{Z_1'Z_2' \cdots Z_N}{R_1'R_2' \cdots R_N}. \tag{11}$$

Apart from currents it is interesting to know the law of change of voltage V across the primary circuit which is also the reactive voltage across the tube:

$$\frac{V_0}{V} = \frac{I_{10}}{I_1} \frac{Z_{M0}}{Z_M} = \sim \frac{I_{10}}{I_1} \frac{\omega_0}{\omega}.$$

Combining this with (9):

$$\frac{V_0}{V} = \frac{1}{1+\beta}\left(\frac{Z_1'}{R_1'} + \beta\right). \tag{9'}$$

Combining (9′) and (10):

$$\frac{V_0}{V} = \frac{I_{N0}/I_N}{\left(\frac{\omega_0}{\omega}\right)^{N-2} \frac{Z_2'Z_3' \cdots Z_N}{R_2'R_3' \cdots R_N}}. \tag{12}$$

The divisor represents the curve of the output current for the case when first circuit on the input end is omitted and the source of e.m.f. is connected in series with the second circuit (see (11)). Consequently, for obtaining the voltage characteristic, the ordinates of (10) should be divided by the ordinates of (11), the latter representing a system from 2 to N circuits.

Similarly, the current curve in the circuit number n:

$$\frac{I_{n0}}{I_n} = \frac{I_{N0}/I_N}{\left(\frac{\omega_0}{\omega}\right)^{N-n} \frac{Z'_{n+1}Z'_{n+2} \cdots Z_N}{R'_{n+1}R'_{n+2} \cdots R_N}}$$

is a ratio of ordinates of the curves of Fig. 10 to ordinates of the curves of Fig. 11 which corresponds to a system from $(n+1)$ to N circuits with the source of e.m.f. connected in series with number $(n+1)$ circuit.

In case of a single circuit (Fig. 5), neglecting the term ω/ω_0, we have:

$$\frac{I_{10}}{I_1} = \frac{1}{1+\beta}\left[\frac{Z_1}{R_1}+\beta\right] = 1 + j\frac{2\Omega}{\omega\delta_1(1+\beta)}.$$

The modulus of this equation is

$$\left|\frac{I_1}{I_{10}}\right| = \frac{1}{\sqrt{1+\left[\frac{2\Omega}{\omega\delta_1(1+\beta)}\right]^2}}. \qquad (14)^*$$

Comparing it with usual single-circuit resonance formula

$$\left|\frac{I_1}{I_{10}}\right| = \frac{1}{\sqrt{1+\left(\frac{2\Omega}{\omega\delta_1}\right)^2}} \qquad (14')$$

we realize that the tube causes an increase of decrement $(1+\beta)$ times, and therefore improves the transmission curve.

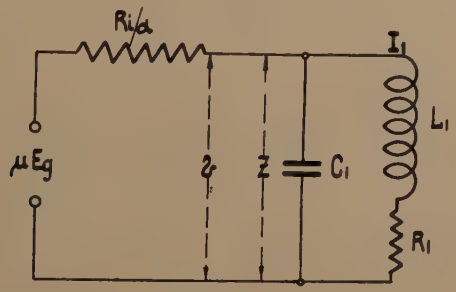

Fig. 5

In case of several circuits, the effect of the tube is to increase the decrement of the primary circuit (Fig. 4) by $\beta\delta_1'$, and to increase the total decrement δ_1', $(1+\beta)$ times.

II. Two-Circuit System

For the two-circuit case (Fig. 6) we have:

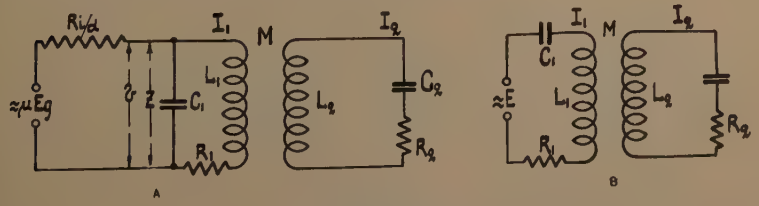

Fig. 6

* The expression was in another way derived by G. A. Zeitlenok, *Wcstnik Elektrotechniki* no. 1, (1930).

$$\frac{I_{20}}{I_2} = \frac{1}{1+\beta}\left(\frac{Z_1'Z_2}{R_1'R_2} + \beta\frac{Z_2}{R_2}\right) \tag{10'}$$

$$\frac{V_0}{V} = \frac{I_{20}/I_2}{Z_2/R_2} \tag{12'}$$

$$R_1'R_2 = R_1R_2 + \omega_0^2M^2 = R_1R_2\left(1 + \frac{K^2}{\delta_1\delta_2}\right) = R_1R_2\frac{\delta_1'}{\delta_2}$$

$$Z_1'Z_2 = Z_1Z_2 + \omega^2M^2$$

$$= R_1R_2\left[\left(1 + j\frac{2\Omega}{\omega\delta_1}\right)\left(1 + j\frac{2\Omega}{\omega\delta_2}\right) + \left(\frac{\omega}{\omega_0}\right)^2\frac{K^2}{\delta_1\delta_2}\right].$$

Neglecting Ω^2/ω_0^2 as compared to 1, we have:

$$\left(\frac{\omega}{\omega_0}\right)^2 \cong 1 + \frac{2\Omega}{\omega_0}.$$

After suitable substitutions, we derive:

$$\frac{Z_1'Z_2}{R_1'R_2} = 1 - \left(\frac{2\Omega}{\omega}\right)^2\frac{1}{\delta_1'\delta_2} + \frac{2\Omega}{\omega_0}\frac{\delta_1' - \delta_1}{\delta_1'} + j\frac{2\Omega}{\omega}\frac{(\delta_1 + \delta_2)}{\delta_1'\delta_2} \tag{15}$$

$$\frac{I_{20}}{I_2} = \frac{1}{1+\beta}\left[1 + \beta - \left(\frac{2\Omega}{\omega}\right)^2\frac{1}{\delta_1'\delta_2} + \frac{2\Omega}{\omega_0}\frac{\delta_1' - \delta_2}{\delta_1'} + j\frac{2\Omega}{\omega}\frac{(\delta_1 + \delta_2 + \beta\delta_1')}{\delta_1'\delta_2}\right]. \tag{16}$$

The modulus of the above expression (which will permit us to plot a resonance curve) is

$$\left|\frac{I_2}{I_{20}}\right| = \frac{1+\beta}{\sqrt{\left[(1+\beta) - \left(\frac{2\Omega}{\omega}\right)^2\frac{1}{\delta_1'\delta_2} + \frac{2\Omega}{\omega_0}\frac{\delta_1' - \delta_1}{\delta_1'}\right]^2 + \left[\frac{2\Omega}{\omega}\frac{(\delta_1 + \delta_2 + \beta\delta_1')}{\delta_1'\delta_2}\right]^2}}. \tag{17}$$

The phase shift against the resonance frequency is

$$\mu = \operatorname{arc\,tg}\frac{\dfrac{2\Omega}{\omega}\dfrac{\delta_1 + \delta_2 + \beta\delta_1'}{\delta_1'\delta_2}}{1 + \beta - \left(\dfrac{2\Omega}{\omega}\right)^2\dfrac{1}{\delta_1'\delta_2} + \dfrac{2\Omega}{\omega_0}\dfrac{\delta_1' - \delta_1}{\delta_1'}}. \tag{18}$$

The expression for amplitudes (17) and phases (18) are asymmetrical, which means that side bands in case of modulation by frequency Ω will not be equally changed as regards amplitude and phase; this will result in additional distortion, due to phase modulation. In ordinary

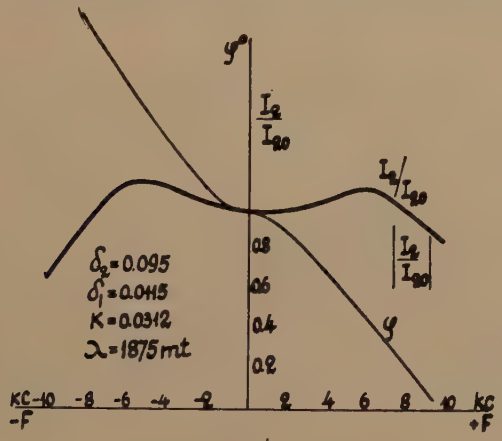

Fig. 7

practice the asymmetry of the amplitude and phase curves is very slight (see Fig. 7), and the average of the values $\omega_0+\Omega$ and $\omega_0-\Omega$ may be used with sufficient accuracy; this allows for a considerable simplification of formulas (17) and (18).

Let:

$$\left|\frac{I_2}{I_{20}}\right| = \mu_2 \cdots \text{demodulation factor},$$

and for convenience let

$$m = \frac{2\Omega}{\omega} = \frac{2F}{f}$$

wherein, F is the audio frequency.

The demodulation factor then becomes

$$\mu_2 = \frac{1}{\sqrt{1+\left[\frac{m_2}{\delta_1'\delta_2(1+\beta)}\right]^2 - \frac{m_2}{\delta_1'\delta_2(1+\beta)}\left[2-\frac{(\delta_1+\delta_2+\beta\delta_1')^2}{\delta_1'\delta_2(1+\beta)}\right]}} \cdot \quad (19)$$

For the case $\beta=0$ (Fig. 6B) we have

$$\mu_2' = \frac{1}{\sqrt{1+\left(\frac{m^2}{\delta_1'\delta_2}\right)^2 - \frac{m^2}{\delta_1'\delta_2}\left(2-\frac{(\delta_1+\delta_2)^2}{\delta_1'\delta_2}\right)}} \cdot \quad (20)$$

Now the modulated current curve is

$$i_2 = I_{20} \sin \omega_0 t + \mu_2 \frac{I_{20} M}{2} \{\cos [(\omega_0 - \Omega)t + \varphi] - \cos [(\omega_0 + \Omega)t - \varphi]\}$$

$$= I_{20} \sin \omega_0 t [1 + \mu_2 M \sin (\Omega t - \varphi)]$$

in which M is the modulation factor for transmission without distortion. Inasmuch as phase displacements φ are not audible, the amplitude distortion μ_2 determined by (19) or (20) is the only factor to be considered. Equation (19) shows that demodulation depends neither on the kind of coupling between circuits, nor on the character of coupling between the plate and the primary circuit. In fact capacitive coupling between the circuits will merely reverse the sign of the term $2\Omega/\omega_0$ $(\delta_1' - \delta_1)/\delta_1'$; the + will take place of the −. This term is relatively small and affects the asymmetry to but a slight degree. For that reason, formulas (17) and (18) can, without appreciable error, be applied to all practical methods of coupling.

In ordinary telephone operation of a transmitter, the factor $\beta = \alpha Z_0/R_i$ does not exceed 0.25, and possible variations of β during plate or grid modulation cannot result in a great distortion of the transmission curve. The factor β can be neglected in rough calculations.

In the case of the output stage of a transmitter, the natural decrement δ_1 of the primary circuit can be neglected, because of the very high value of efficiency of that circuit. It is convenient here to express the coupling factor in terms of decrement δ_2

$$K = A\delta_2 \tag{21}$$

wherein A is a definite coefficient. Therefore,

$$\delta_1' = \delta_1 + A^2\delta_2 \cong A^2\delta_2$$

and (19) becomes

$$\mu_2 = \frac{1}{\sqrt{1 + \left(\frac{m}{\delta_2}\right)^2 \frac{1}{A^4(1+\beta)^2} \left[\left(\frac{m}{\delta_2}\right)^2 - 2A^2 + A^4\beta^2 + 1\right]}} \tag{19'}$$

If μ_2 is plotted as a function of m/δ_2, the characteristics will be entirely dependent on the given A and β, and the transmission curves at various waves or decrements δ_2 will differ but by the scale of the abscissas. (Figs. 8 and 9.)

Now, let us consider the shape of transmission curve (19). For the case:

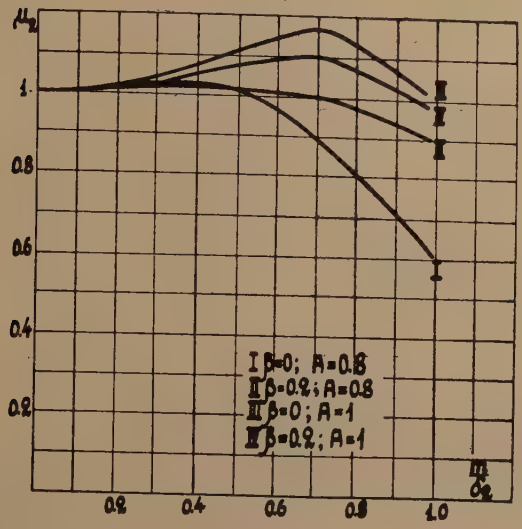
μ2
1
0.8
0.6
0.4
0.2
I β=0; A=0.8
II β=0.2; A=0.8
III β=0; A=1
IV β=0.2; A=1
m/δ2
0.2
0.4
0.6
0.8
1.0

Fig. 8

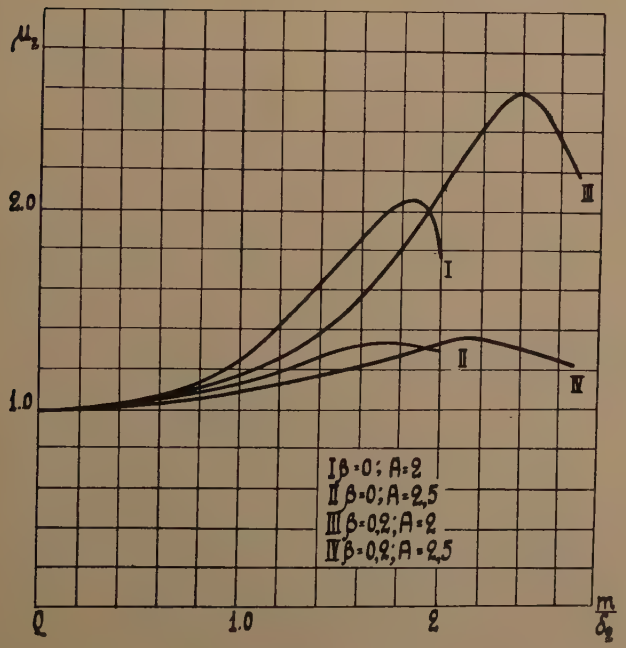
μ2
2.0
1.0
I β=0; A=2
II β=0; A=2,5
III β=0,2; A=2
IV β=0,2; A=2,5
m/δ2
0
1.0
2

Fig. 9

$$2 - \frac{(\delta_1 + \delta_2 + \beta\delta_1')^2}{\delta_1'\delta_2(1+\beta)} < 0$$

the transmission curve will be single humped.

For the case:

$$2 - \frac{(\delta_1 + \delta_2 + \beta\delta_1')^2}{\delta_1'\delta_2(1+\beta)} > 0$$

the transmission curve will be double humped.

The boundary condition:

$$2 - \frac{(\delta_1 + \delta_2 + \beta\delta_1')^2}{\delta_1'\delta_2(1+\beta)} = 0$$

corresponds to the so-called "critical coupling".

For either method of application of e.m.f., the following expression for the critical coupling can be adopted:

$$K_N \leqq \sqrt{(\delta_1^2 + \delta_2^2)/2}. \tag{22}$$

In case of double-humped curves for μ_2 (19) and μ_2' (20), the value of $\mu_{2\max}$ becomes interesting. Differentiating (19) we find that the maximum occurs when:

$$m^2 = \delta_1'\delta_2(1+\beta) - \frac{(\delta_1 + \delta_2 + \beta\delta_1')^2}{2} \tag{23}$$

whence,

$$\mu_{2\max} = \frac{\delta_1'\delta_2(1+\beta)}{(\delta_1 + \delta_2 + \beta\delta_1')} \cdot \frac{1}{\sqrt{\delta_1'\delta_2(1+\beta) - \frac{(\delta_1 + \delta_2 + \beta\delta_1')^2}{4}}}. \tag{24}$$

Substituting (21) we find:

$$\mu_{2\max} \leqq \frac{(1+\beta)A^2}{(1+A^2\beta)\sqrt{A^2 - \frac{(1-A^2\beta)^2}{4}}}. \tag{24'}$$

For the case of reception (20) the maximum occurs when

$$m = \sqrt{K^2 - (\delta_1^2 + \delta_2^2)/2} \tag{25}$$

and,

$$\therefore \quad \mu'_{2\max} = \frac{\delta_1\delta_2 + K^2}{(\delta_1 + \delta_2)\sqrt{K^2 - (\delta_1 - \delta_2)^2/4}}. \tag{26}$$

It is seen from comparison of (23) with (25) and (24) with (26) that when a tube is used the maximum occurs at somewhat lower frequency, and that the value of the maximum is lower. Figs. 9 and 10 show that the effect of a tube is to flatten the transmission curve.

Equation (19) makes it possible to predetermine the degree of coupling necessary for undistorted transmission of frequency F' as well as $n'=2\Omega'/\omega_0$ with a given decrement of the secondary circuit, δ_2. Thus from (19′)

$$A^2 = \frac{1 \pm \sqrt{1 - [1 + (m/\delta_2)^2]\beta^2}}{\beta^2} . \tag{27}$$

For $\beta=0$

$$A^2 = \frac{m^2 + \delta_1^2 + \delta_2^2}{2\delta_2^2} . \tag{27'}$$

The above leads to the conclusion that a tank circuit can bring about a transmission of frequencies that cannot be obtained with a single circuit owing to its drooping resonance curve, and that effective transmission of frequencies can be accomplished at relatively slight decrement of the secondary circuit. This is true for the case of reception if the asymmetry of the transmission curve is not taken into account. In case of transmission it is true only as long as the plate reaction does not exceed a certain limit. When the plate reaction is great enough to produce dips in the plate current characteristic (or to make it saddle-shaped) the transmission of frequencies quickly becomes poor. From (12′) the magnitude of plate reaction for the two-circuit system can be determined as

$$\frac{V}{V_0} = \frac{1 + j\dfrac{2\Omega}{\omega\delta_2}}{1 - \left(\dfrac{2\Omega}{\omega}\right)^2 \dfrac{1}{\delta_1'\delta_2(1+\beta)} + j\dfrac{2\Omega}{\omega}\dfrac{(\delta_1 + \delta_2 + \beta\delta_1')}{\delta_1'\delta_2(1+\beta)}} .$$

Let γ = modulus of the above equation; i.e.,

$$\gamma = \left|\frac{V}{V_0}\right| = \frac{|I_2/I_{20}|}{\sqrt{1 + \left(\dfrac{2\Omega}{\omega\delta_2}\right)^2}} . \tag{28}$$

In other words, the voltage curve is the ratio of ordinates of resonance curve $|I_2/I_{20}|$ to those of the resonance curve of an isolated

secondary circuit (Fig. 10). Barring the asymmetry, we have:

$$\gamma^2 = \mu_2^2\, 1\left[+(m/\delta_2)^2\right] \tag{29}$$

or, substituting from (19)

$$\gamma^2 = \frac{1 + \left(\frac{m}{\delta_2}\right)^2}{1 + \left(\frac{m}{\delta_2}\right)^2 \frac{1}{A^4(1+\beta)^2}\left[\left(\frac{m}{\delta_2}\right)^2 - 2A^2 + A^4\beta^2 + 1\right]}. \tag{29'}$$

It is evident that the same law governs the primary circuit current characteristic $|I_1/I_{10}| = f(\omega)$; its divergence from the voltage characteristic (8) being only in the straight-line factor ω/ω_0; this factor is

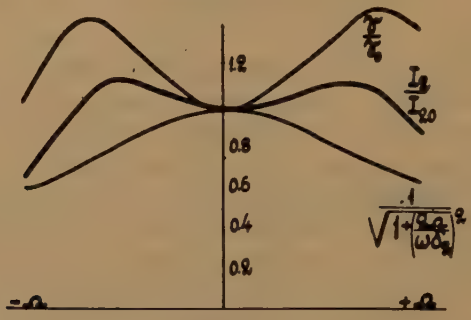

Fig. 10

neglected by assuming the characteristics to be symmetrical. There is no necessity to take the resonance curve of the secondary circuit (antenna) for the determination of decrement δ_2; the curve can be determined as the ratio of ordinates of curve $|I_2/I_{20}|$ to ordinates of curve $|I_1/I_{10}|$ and plotted. Since the resonance curve of one circuit, δ_2, is always drooping with maximum at $\omega = \omega_0$, the voltage curve (28), or primary current curve, as a rule, should have a more pronounced double-humped character than current curve (19). With suitable coupling factors, current, I_2/I_{20}, curves having no humps are obtainable, while the voltage, V/V_0, curve will be sharply double humped.

By making $\gamma = 1$ in (29), the frequency range of $V > V_0$ can be obtained. Thus

$$m_1 = \frac{2\Omega_1}{\omega_0} \leqq \delta_2 \sqrt{A^4(1+2\beta) + 2A^2 - 1}. \tag{30}$$

Since the primary and secondary circuits have different transmission curves, it is necessary, when measuring frequencies of the trans-

mitter, to keep the instrument that measures percentage modulation coupled only to the secondary circuit (antenna) and not to the primary.

Let,

$$\frac{V_0}{E_p} = 0.9$$

wherein E_p = supply voltage.

Taking the resonance curve of a generator operating at such V_0 on two circuits we should have for some part of the frequency range $V > v_0$ and possibly $V/E_p > 1$. Actually, due to valleys or dips in plate-current curve the voltage across the circuit will be lower than that given by (29). The values of current, I_2/I_{20}, will be proportionally lower. Hence, the exact calculated values can be expected only when there is no overvoltage on the plate; i.e., when $V \leqq V_{\text{lim}} < E_p$.

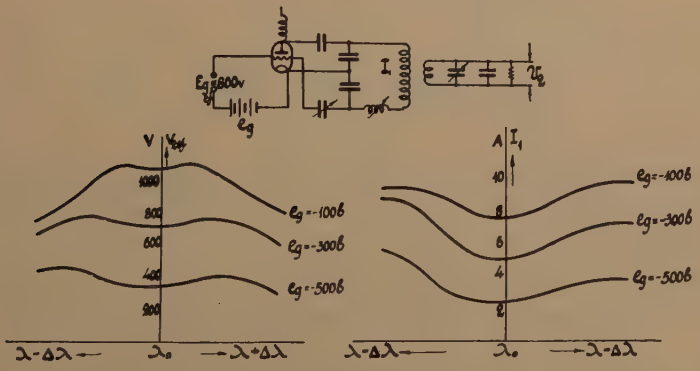

Fig. 11

Fig. 11 shows a set of resonance curves taken at different values of V_0. At high values of V_0 the curves are distorted. During modulation, due to overvoltages the frequency curves also will be distorted. The distortion increases with increase of percentage modulation. For example, let the transmitter be set for 100 per cent modulation, then it will have $V_{0t} = V_0/2 = (0.4-0.45)E_p$ without modulation. During modulation:

$$V = V_{0t}(1 + M\gamma).$$

When the modulation factor M is low and $V_{\max}$ can be lower than $V_{\text{lim}} < E_p$, the transmission curve will not be distorted. When $V_{\max} > V_{\text{lim}}$ the curves will be distorted and the distortion will increase with increase of M. This is proved by actual transmission curves, taken at

various modulation factors. (Fig. 12.) Hence, we come to the conclusion that an efficiently utilized generator (as regards power and modulation factor) has very limited abilities as regards improved transmission of frequencies by means of a tank circuit.

Since the calculation of overexcited generator operation is pretty complicated, while undistorted modulation is possible in underexcited conditions only, the complicated calculation of distorted transmission

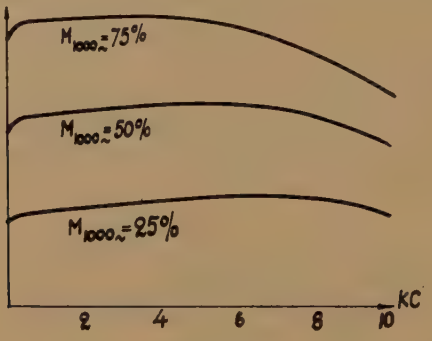

Fig. 12

curve is not warranted in practice. For evaluating voltages possible one can determine γ_{max} under given conditions. Analyzing (29′) we obtain γ_{max} at

$$m = \delta_2\sqrt{A\sqrt{A^2(1+2\beta)+2}-1} \tag{31}$$

and,

$$\gamma^2_{max} = \frac{A^4(1+\beta)^2\sqrt{A^2(1+2\beta)+2}}{2A^3(1+2\beta)+4A-(2A^2+1-A^4\beta^2)\sqrt{A^2(1+2\beta)+2}}. \tag{32}$$

Fig. 13 shows the values of γ_{max} plotted against A for various values of β. The graph shows the decrease of overvoltages with increase of β.

From (29) the mode of change of voltage phase can be determined. There is no phase displacement at $m=0$ and

$$m = \frac{2\Omega}{\omega_0} = \pm\sqrt{K^2-\delta_2^2} = \pm\delta_2\sqrt{A^2-1}. \tag{33}$$

At three frequencies other than resonance frequency, therefore, there will be no phase displacement, and Z becomes pure resistance.

When $k \leqq \delta_2$ phase displacement will be zero at resonance frequency, ω_0 only.

The ratio of power delivered to the secondary circuit at frequency ω to power delivered at resonance is ω_0

$$\frac{P_2}{P_{20}} = \mu_2^2.$$

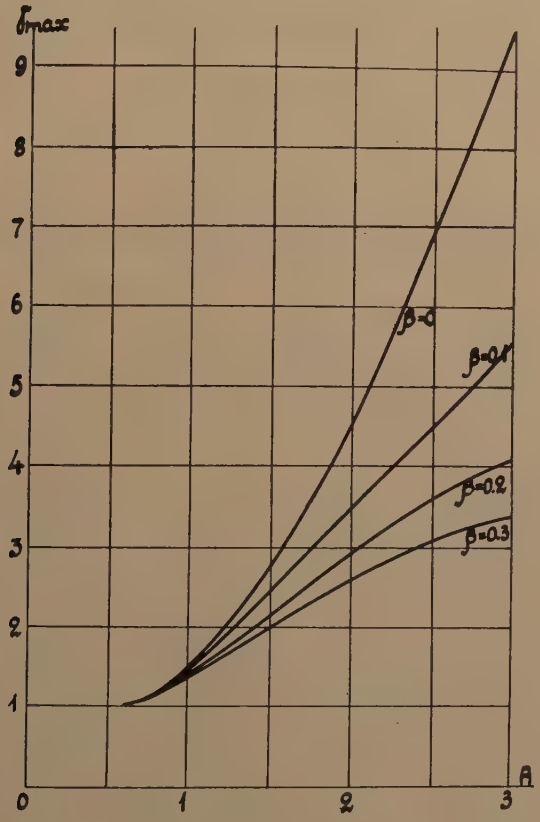

Fig. 13

The ratio of power, lost in the primary circuit at frequency ω to the power lost at resonance,

$$\frac{P_1}{P_{10}} = \gamma^2.$$

The supplied generator output is dependent on the frequency in the following way:

$$\frac{P}{P_0} = \frac{\mu_2^2 P_{20} + \gamma^2 P_{10}}{P_{20} + P_{10}}.$$

Let $\eta = P_2/P$, efficiency of the tank circuit, then

$$\frac{P}{P_0} = \mu_2^2\eta_0 + \gamma^2(1 - \eta_0) \leqq \mu_2^2$$

i.e., with double-humped characteristic the generator output over a certain part of the frequency range is greater than the output at resonance frequency.

The variation of the tank circuit efficiency is:

$$\frac{\eta_0}{\eta} = \eta_0 + \frac{\gamma^2}{\mu_2^2}(1 - \eta_0).$$

Hence, the greatest efficiency of tank circuit occurs at carrier frequency.

All the foregoing conclusions are based on the assumption that each circuit is tuned to the carrier frequency. It may happen in practice that the secondary circuit is not tuned to carrier frequency, but the resistance it introduces in the primary is compensated for by suitable tuning of the primary, so that the whole system acts as active load on the generator at carrier frequency, Z_0. (It may also happen that the circuits are tuned to one of the frequencies, $\omega_0 + \Omega$, according to (33), when Z is a pure resistance load. In the latter case the resonance curve will be asymmetrical, and either negative or positive side bands will pass easier than others. Such a tuning is not likely to take place, however, for it is too complicated.) Inaccuracy of tuning the secondary circuit compensated by proper tuning of primary is very probable in practice, since the tuning of the whole system for resistive load is easily determined by plate and grid meter readings; at the same time the current in the secondary, when this is inaccurately tuned, can be even heavier than when the secondary is exactly tuned.

The disadvantages of an inaccurate tuning will be shown later on. For that reason, we confine ourselves to consideration of a simpler case, when the e.m.f. is directly applied to the primary (Fig. 6B). Since the circuits are not tuned to the carrier frequency, the impedances Z_{10} and Z_{20} should be substituted for the resistances R_1' and R_2 in (11′) which determine the resonance curve I_{20}/I_2. Hence,

$$\frac{I_{20}}{I_2} = \frac{\omega_0}{\omega} \frac{Z_1'Z_2}{Z_{10}'Z_{20}}.$$

Suppose that the circuits were previously tuned to carrier frequency; i.e.,

$$\omega_0^2 = \frac{1}{L_1 C_1} = \frac{1}{L_2 C_2}$$

and thereupon the inductance of the secondary was changed by $\Delta_2 L_2$. The secondary impedance at carrier frequency is

$$Z_{20} = R_2 + j\omega_0 \Delta_2 L_2$$

and the reactance

$$-j \frac{\omega_0^2 M^2}{R_2^2 + (\omega_0 \Delta_2 L_2)^2} \omega_0 \Delta_2 L_2$$

is inserted in the primary. This reactance is compensated for by the change in inductance $\Delta_1 L_1$, so that,

$$\omega_0 \Delta_1 L_1 = \frac{\omega_0^2 M^2}{R_2 \left[1 + \frac{(\omega_0 \Delta_2 L_2)^2}{R_2}\right]} \frac{\omega_0 \Delta_2 L_2}{R_2}.$$

Denote as before,

$$\delta_1 = \frac{R_1}{\omega_0 L_1}; \quad \delta_2 = \frac{R_2}{\omega_0 L_2}; \quad K = \frac{M}{\sqrt{L_1 L_2}}; \text{ etc.},$$

hence,

$$\frac{\omega_0 \Delta_1 L_1}{R_1} = \frac{\Delta_1}{\delta_1} = \frac{\Delta_2}{\delta_2} \frac{K^2}{\delta_1 \delta_2 \left(1 + \frac{\Delta_2^2}{\delta_2^2}\right)}$$

$$\Delta_1 = \Delta_2 \frac{K^2}{\delta_2^2 + \Delta_2^2}$$

$$Z_{10}' Z_{20} = R_1 R_2 \left[\frac{\delta_1'}{\delta_1} - \frac{\Delta_1 \Delta_2}{\delta_1 \delta_2} + j\left(\frac{\Delta_1}{\delta_1} + \frac{\Delta_2}{\delta_2}\right)\right].$$

Denote the natural frequencies of circuits:

$$\omega_1 = \frac{1}{L_1 C_1 (1 + \Delta_1)} \leqq \left(1 - \frac{\Delta_1}{2}\right) \omega_0$$

$$\omega_2 \leqq \left(1 - \frac{\Delta_2}{2}\right) \omega_0.$$

Then,

$$\omega = \omega_0 + \Omega = \omega_1 + \alpha_1 = \omega_2 + \alpha_2$$

$$\alpha_1 \cong \frac{\Delta_1}{2}\omega_0 + \Omega$$

$$\alpha_2 \cong \frac{\Delta_2}{2}\omega_0 + \Omega$$

$$Z_1 \cong R_1\left[1 + j\frac{2\alpha_1}{\omega\delta_1}\sqrt{1+\Delta_1}\right]$$

$$Z_2 \cong R_2\left[1 + j\frac{2\alpha_2}{\omega\delta_1}\sqrt{1+\Delta_2}\right]$$

$$Z_1'Z_2 = Z_1Z_2 + \omega^2M^2 = R_1R_2[S + jT]$$

and,

$$\left|\frac{I_2}{I_{20}}\right|^2 = \left(\frac{\omega}{\omega_0}\right)^2 \frac{\left[\frac{\delta_1'}{\delta_1} - \frac{\Delta_1\Delta_2}{\delta_1\delta_2}\right]^2 + \left[\frac{\Delta_1}{\delta_1} + \frac{\Delta_2}{\delta_2}\right]^2}{S^2 + T^2} \tag{34}$$

in which,

$$\left.\begin{aligned} S &= \left[1 + \left(\frac{\omega}{\omega_0}\right)^2 \frac{\delta_1' - \delta_1}{\delta_1} - \frac{4}{\delta_1\delta_2}\left(\frac{\omega_0}{\omega}\right)^2\left(1 + \frac{\Delta_1 + \Delta_2}{2}\right)\right. \\ &\qquad \left.\left(\frac{\Delta_1\Delta_2}{2} + \frac{\Omega}{\omega_0}\,\frac{\Delta_1 + \Delta_2}{2} + \frac{\Omega^2}{\omega^2}\right)\right] \\ T &= \frac{2\omega_0}{\omega}\left[\frac{\Delta_1}{2\delta_1} + \frac{\Delta_2}{2\delta_2} + \frac{\Omega}{\omega_0}\left(\frac{\sqrt{1+\Delta_1}}{\delta_1} + \frac{\sqrt{1+\Delta_2}}{\delta_2}\right)\right] \end{aligned}\right\} \tag{34'}$$

(Certain factors in these equations can be neglected depending upon the conditions given.)

Fig. 14 shows resonance curves for various degrees of detuning, Δ_2. Similar results are obtained from experiments; the resonance curves are asymmetrical with respect to frequency ω_0 and therefore the modulation will be distorted due to phase modulation. It should be mentioned that by slightly detuning it is possible to eliminate the small asymmetry that exists when circuits are exactly tuned. Meters, however, do not give any preliminary indication as to the amount of this detuning—the symmetry of the curve can be ascertained only after having taken the curve itself.

When controlling a modern powerful transmitter having several stages of amplification of modulated waves, particular attention should

be paid to the accuracy of tuning the various circuits. Otherwise the resulting transmission curve may be quite unsatisfactory, in spite of the correct design of circuits of individual stages.

Inaccurate tuning cannot be recommended also because there is no definite criterion for it under practical conditions, particularly in such cases where the secondary is represented by an antenna, the parameters of which are subject to certain variations (due to sleet, etc.).

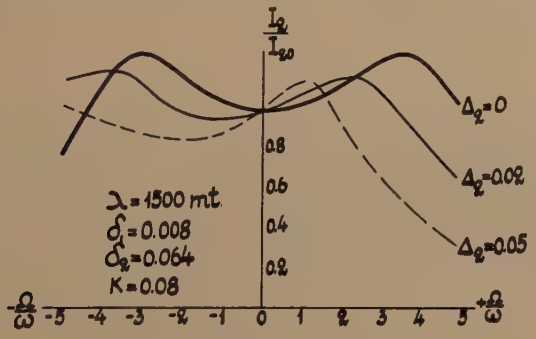

Fig. 14

The proper tuning, therefore, is when the secondary is exactly tuned to carrier frequency and the primary which is coupled to the secondary is tuned for resistive load (careful neutralization is absolutely necessary).

Under operating conditions the accuracy of tuning can be fairly exactly checked by readings of the plate and grid direct-current meters, since when tuned to resonance the secondary circuit introduces the maximum resistance $\omega_0^2 M^2/R_2$ into the primary. At this point the impedance Z_0 of generator load becomes minimum and, therefore, the minimum of plate current becomes the highest.

III. Three-Circuit System

The three-circuit system is shown in Fig. 15. Similarly to (10), (12), and (13) we write

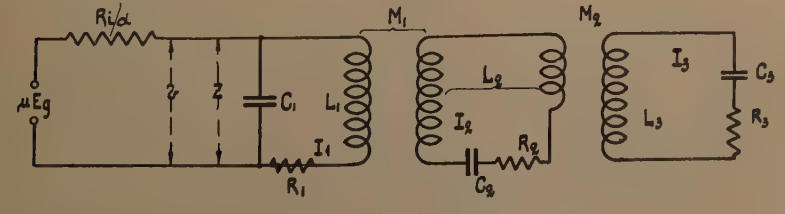

Fig. 15

$$\frac{I_{30}}{I_3} = \frac{\omega_0}{\omega}\,\frac{1}{1+\beta}\left(\frac{Z_1'Z_2'Z_3}{R_1'R_2'R_3} + \beta\frac{Z_2'Z_3}{R_2'R_3}\right) \tag{10''}$$

$$\frac{V_0}{V_3} = \frac{I_{30}/I_3}{\dfrac{\omega_0}{\omega}\,\dfrac{Z_2'Z_3}{R_2'R_3}} \tag{12''}$$

$$\frac{I_{20}}{I_2} = \frac{I_{30}/I_3}{\dfrac{\omega_0}{\omega}\,\dfrac{Z_3}{R_3}}. \tag{13''}$$

Here,

$$Z_1'Z_2'Z_3 = Z_1Z_2Z_3 + Z_1\omega^2M_2^2 + Z_3\omega^2M_1^2$$
$$R_1'R_2'R_3 = R_1R_2R_3 + R_1\omega_0^2M_2^2 + R_3\omega_0^2M_1^2.$$

Making computations similar to those for the two-circuit system, we obtain

$$Z_1'Z_2'Z_3 = R_1R_2R_3(T + jU)$$

in which,

$$T = 1 - \left(\frac{2\Omega}{\omega}\right)^2 \frac{\delta_1+\delta_2+\delta_3}{\delta_1\delta_2\delta_3} + \left(\frac{\omega}{\omega_0}\right)^2 \frac{K_1^2\delta_3 + K_2^2\delta_1}{\delta_1\delta_2\delta_3}$$

$$U = \frac{2\Omega}{\omega\delta_1}\left[1 - \left(\frac{2\Omega}{\omega}\right)^2 \frac{1}{\delta_2\delta_3} + \left(\frac{\omega}{\omega_0}\right)^2 \frac{K_1^2+K_2^2}{\delta_2\delta_3} + \frac{\delta_1(\delta_2+\delta_3)}{\delta_2\delta_3}\right]$$

$$R_1'R_2'R_3 = R_1R_2R_3\left[1 + \frac{K_1^2\delta_3 + K_2^2\delta_1}{\delta_1\delta_2\delta_3}\right].$$

Since,

$$\delta_1' = \delta_1 + \frac{K_1^2}{\delta_2'};\quad \delta_2' = \delta_2 + \frac{K_2^2}{\delta_3}$$

then,

$$1 + \frac{K_1^2\delta_3 + K_2^2\delta_1}{\delta_1\delta_2\delta_3} = \frac{\delta_1'\delta_2'}{\delta_1\delta_2}.$$

Hence:

$$\frac{Z_1'Z_2'Z_3}{R_1'R_2'R_3} = 1 - \left(\frac{2\Omega}{\omega}\right)^2 \frac{\delta_1+\delta_2+\delta_3}{\delta_1'\delta_2'\delta_3} + \frac{2\Omega}{\omega_0}\,\frac{K_1^2\delta_3 + K_2^2\delta_1}{\delta_1'\delta_2'\delta_3}$$
$$+ j\frac{2\Omega}{\omega\delta_1'}\left[\frac{\delta_1\delta_2 + \delta_1\delta_3 + \delta_2\delta_3}{\delta_2'\delta_3} - \left(\frac{2\Omega}{\omega}\right)^2 \frac{1}{\delta_2'\delta_3}\right.$$

$$+\left(\frac{\omega}{\omega_0}\right)^2 \frac{K_1^2+K_2^2}{\delta_2'\delta_3}\Bigg] \tag{35}$$

$$\frac{Z_2'Z_3}{R_2'R_3} = 1-\left(\frac{2\Omega}{\omega}\right)^2 \frac{1}{\delta_2'\delta_3} + \frac{2\Omega}{\omega_0}\frac{\delta_2'-\delta_2}{\delta_2'} + j\frac{2\Omega}{\omega}\frac{\delta_2+\delta_3}{\delta_2'\delta_3}. \tag{36}$$

Substituting (35) and (36) in (10″), (12″), and (13″) the exact solutions can be obtained.

Since the efficiencies of actual transmitter circuits are very high, and consequently,

$$\delta_1 \ll \delta_1'; \quad \delta_2 \ll \delta_2'.$$

The above expressions can, without considerable error, be simplified by neglecting the decrements of the primary and secondary circuits (δ_1 and δ_2); i.e.,

$$\frac{K_1^2}{\delta_2'} = \delta_1' - \delta_1 \cong \delta_1'; \quad \frac{K_2^2}{\delta_3} = \delta_2' - \delta_2 \cong \delta_2'.$$

Under these assumptions

$$\frac{K_1^2\delta_3 + K_2^2\delta_1}{\delta_1'\delta_2'\delta_3} \cong 1; \quad \frac{K_1^2+K_2^2}{\delta_2'\delta_3} \cong 1+\frac{\delta_1'}{\delta_3}.$$

Further, neglecting $2\Omega/\omega_0 \ll 1$

$$\frac{Z_1'Z_2'Z_3}{R_1'R_2'R_3} \cong 1-\left(\frac{2\Omega}{\omega}\right)^2 \frac{\delta_1+\delta_2+\delta_3}{\delta_1'\delta_2'\delta_3} + j\frac{2\Omega}{\omega\delta_1'}\left[1+\frac{\delta_1'}{\delta_3}-\left(\frac{2\Omega}{\omega}\right)^2\frac{1}{\delta_2'\delta_3}\right] \tag{35'}$$

$$\frac{Z_2'Z_3}{R_2'R_3} \cong 1-\left(\frac{2\Omega}{\omega}\right)^2 \frac{1}{\delta_2'\delta_3} + j\left(\frac{2\Omega}{\omega}\right)\frac{\delta_2+\delta_3}{\delta_2'\delta_3}. \tag{36'}$$

Substituting (35′) and (36′) in (10″):

$$\frac{I_{30}}{I_3} = \frac{\omega_0}{\omega}\frac{1}{1+\beta}\Bigg\{1+\beta-\left(\frac{2\Omega}{\omega}\right)^2\frac{1}{\delta_2'\delta_3}\left[\frac{\delta_1+\delta_2+\delta_3}{\delta_1'}+\beta\right] + j\frac{2\Omega}{\omega\delta_1'}\left[1+\frac{\delta_1'}{\delta_3}-\left(\frac{2\Omega}{\omega}\right)^2\frac{1}{\delta_2'\delta_3}+\beta\frac{\delta_1'(\delta_2+\delta_3)}{\delta_2'\delta_3}\right]\Bigg\}. \tag{37}$$

From this, the modulus of the ratio $|I_3/I_{30}|$, the phase shift angle, etc., can be determined. Barring the asymmetry which, as in a two-circuit case, is very slightly dependent on the method of coupling, we obtain the equation for the demodulation factor:

$$\mu_3 = \frac{1+\beta}{\sqrt{\left[1+\beta-\frac{m^2}{\delta_2'\delta_3}\left(\frac{\delta_1+\delta_2+\delta_3}{\delta_1'}+\beta\right)\right]^2 + \frac{m^2}{\delta_1'^2}\left[1+\frac{\delta_1'}{\delta_3}-\frac{m^2}{\delta_2'\delta_3}+\beta\frac{\delta_1'(\delta_2+\delta_3)}{\delta_2'\delta_3}\right]^2}} \quad (38)$$

in which, as before,

$$m = \frac{2\Omega}{\omega_0}.$$

In case of reception $\beta = 0$. Barring the asymmetry we get:

$$\mu_3' = \frac{1}{\sqrt{\left[1+m^2\frac{(\delta_1+\delta_2+\delta_3)}{\delta_1'\delta_2'\delta_3}\right]^2 + \frac{m^2}{(\delta_1'\delta_2'\delta_3)^2}[\delta_1\delta_2+\delta_1\delta_3+\delta_2\delta_3-m^2+K_1^2+K_2^2]}} \quad (39)$$

For the preliminary judgment of the shape of transmitter frequency curve δ_1 and δ_2 (38) can be neglected as compared to δ_3, and β can be assume equal to zero. Using similar notations to those previously employed, we have

$$K_2 = A\delta_3; \quad K_1 = B\delta_2'.$$

Therefore,

$$\delta_2' \leqq A^2\delta_3; \quad \delta_1' = A^2B^2\delta_3.$$

If in addition we introduce the quantity (x) defined by

$$x = \frac{m}{\delta_3}$$

we will have,

$$\mu_3'' = \frac{1}{y} \leqq \frac{1}{\left[1-\frac{x^2}{A^4B^2}\right] + \frac{x^2}{A^4B^4}\left[1-\frac{x^2}{A^2}+A^2B^2\right]^2}. \quad (40)$$

Now let

$$a = \frac{3}{A^8B^4}; \quad b = \tfrac{1}{3}(2A^2 + 2A^4B^2 - 1);$$

$$c = \frac{A^4}{3}(1 + 2A^2B^2 - 2B^2 + A^4B^4)$$

$$y = \frac{a}{3}\left(x^6 - 3bx^4 + 3cx^2 + \frac{3}{a}\right).$$

By analyzing the latter function we obtain five points of maximum values:

$$x_1 = 0$$

$$x_{2,3} = \pm\sqrt{b - \sqrt{b^2 - c}}$$

$$x_{4,5} = \pm\sqrt{b + \sqrt{b^2 - c}}.$$

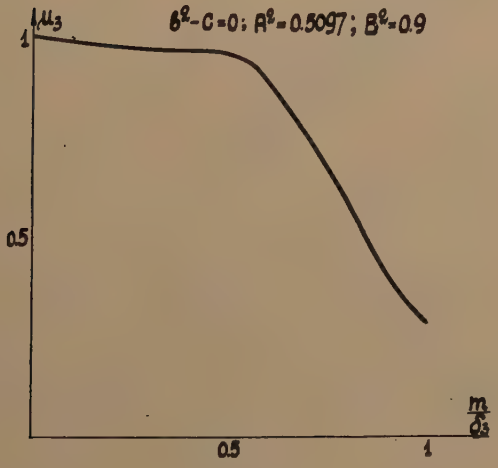

Fig. 16

In the case when all the solutions are real, the curve has three humps, in the case when two solutions, $x_{4,5}$, are real the curve is double humped.

It is easy to see, that when $b^2 - c = 0$ the curve has one extremity only at $x = 0$ and an inflection point at $x = \pm\sqrt{b}$; i.e., the curve is single humped. Then,

$$B^2 = \frac{A\sqrt{b} - 1 - A^2}{A^4}.$$

Such a curve is shown in Fig. 16.

When $b=0$ then,

$$B^2 = \frac{1 - 2A^2}{2A^4}$$

and the curve is either single or double humped, depending upon the sign before c. When $c>0$ the curve is single humped and when $c<0$

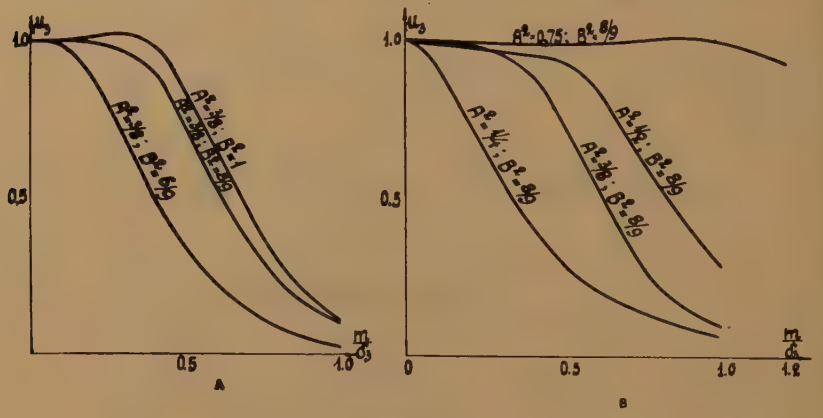

Fig. 17

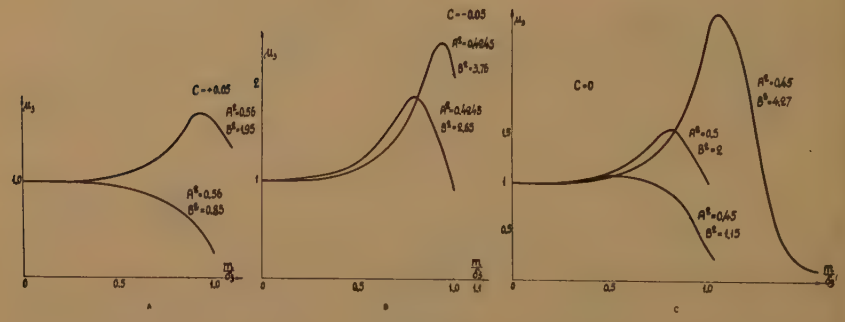

Fig. 18

the curve is double humped. When $b=0$ and $c=0$ a threshold between single and double humped shapes results. Only one value of A and one of B corresponds to that condition (Fig. 17a and 17b):

$$A^2 = 3/8; \quad B^2 = 8/9.$$

When $b>0$ the shape of the curve depends upon the sign before c.

If $c>0$, the curve is either single or triple humped (Fig. 18a). If $c<0$ the curve is double humped (Fig. 18b). If $c=0$, then

$$B^2 = \frac{1 - A^2 \pm \sqrt{1 - 2A^2}}{A^4}$$

and the curve is still double humped with an easy slope at lower modulating frequencies (Fig. 18c).

By means of the graphs shown in Figs. 19 and 20 the shape of a triple-humped curve can be determined; these graphs give the position and magnitude of μ_{3max} and μ_{3min} as dependent on A and B.

The previously stated suggestions in the case of the two-circuit system as to the necessity of accurate tuning which ensures the symmetry

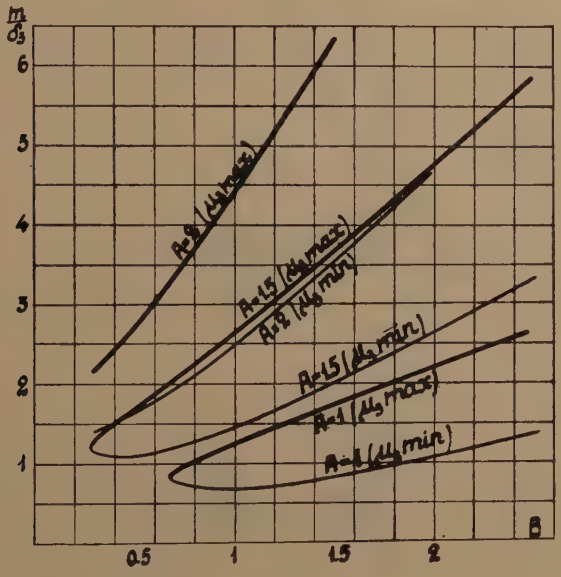

Fig. 19

of the curve and the effect of overvoltages are true for a three-circuit system as well. For proper shape of curves stray coupling between the first and the third circuits should be eliminated.

For overvoltages the following expression serves:

$$\gamma = \mu_3 \sqrt{1 + \left(\frac{m^2}{\delta_2' \delta_3}\right)^2 - \frac{m^2}{\delta_2' \delta_3}\left[2 - \frac{(\delta_2 + \delta_3)^2}{\delta_2' \delta_3}\right]} \quad . \quad . \quad (41)$$

The three-circuit system has certain advantages over the two-circuit system as regards overvoltage. In fact, γ equals the ratio of μ_3 to the ordinates of the transmission curve of the system comprising the

second and the third circuits. The latter curve is double humped when the coupling $k_2 = A\delta_3$ is close enough, and consequently lies below the curve μ_2 (Fig. 21.)

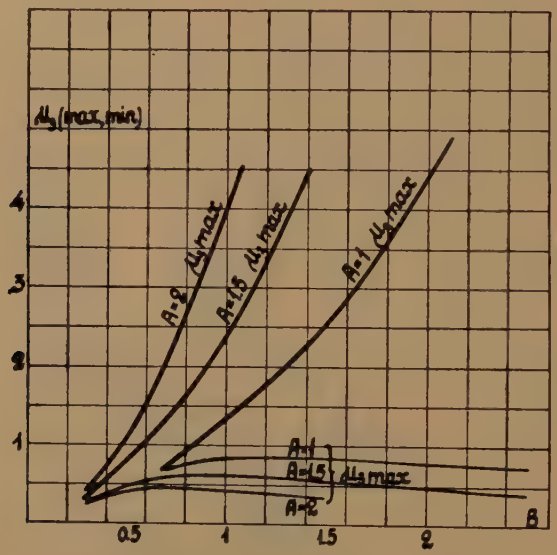

Fig. 20

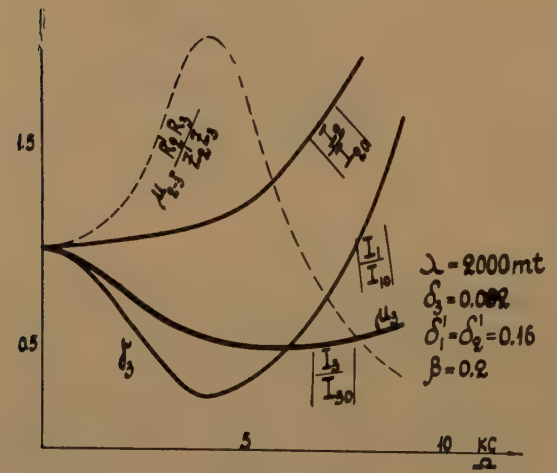

Fig. 21

If the degree of coupling is properly chosen overvoltages can be avoided and a satisfactory transmission curve can be obtained; normally a high value of A and low value of B are required for this condition.

The current curve of the second circuit (Fig. 21) is

$$\left| \frac{I_2}{I_{20}} \right| = \frac{|I_3/I_{30}|}{|R_3/Z_3|} \leqq \mu_3 \sqrt{1 + \frac{m^2}{\delta_3^2}} \cdot \tag{42}$$

When taking the transmission curve care should be taken that the instrument measuring the percentage modulation is coupled with the last circuit only.

The statements made above are confirmed by experimental study of actual networks.

The author wishes to express appreciation of the help received from his assistants in the laboratory.

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EQUIVALENT CIRCUITS OF AN ACTIVE NETWORK*

By

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Summary—*The problem considered in this paper is that of setting up an exactly equivalent circuit for a four-terminal network containing one e.m.f. and any number of impedances connected in any manner. Several relatively simple circuits which can be used to represent any such active network, and some others which can be used in special cases, are completely specified.*

A FOUR-TERMINAL electric network containing no e.m.f.'s is a passive transducer and is sometimes called a passive quadripole. As is well known any passive quadripole may be exactly simulated, as far as steady-state reactions at a given frequency are concerned, by an equivalent T or other simple section so that no external electrical measurement of a steady-state reaction at the given frequency can distinguish between the original quadripole and its equivalent section. Campbell[1] showed this and gave the necessary equations for specifying the T section in terms of the properties[2] of the network. The conditions to be satisfied by the network in order that the Campbell equations hold are simply those which must be satisfied in order that the usual circuit equations in complex numbers may be used for the network, viz., all circuit parameters (resistances, inductances, and capacitances) must be constant at a given frequency; the currents and voltages of one frequency must be treated separately from those of any other; and only steady-state conditions are considered.

There are no e.m.f.'s within a passive quadripole. The simplest case of an active quadripole[3] is that in which there is one e.m.f., and the problem considered here is the determination of a simple equivalent

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[1] G. A. Campbell, "Cisoidal oscillations," *Trans. A.I.E.E.*, vol. 30, p. 873, (1911). The equivalence is with respect to two pairs of terminals, and does not hold with respect to all six possible pairs of terminals.

[2] By "properties" are meant any three of the following quantities, provided the three chosen are independent: short-circuit, open-circuit or other input impedances measured at either pair of terminals of the quadripole, transfer impedances under specified conditions, current ratios, iterative or image impedances, transfer constant, etc.

[3] A passive quadripole is a passive four-terminal transducer, but according to I.R.E. 1931 Standardization Report (No. 1049) an active quadripole is not necessarily an active transducer. The term quadripole has been used here because of its greater generality. When the sources of energy within an active quadripole are controlled by external voltages, the former is equivalent to a four-terminal active transducer.

circuit for an active quadripole which comprises a network containing one alternating e.m.f. and any number and arrangement of impedances. The same conditions as in the case of the passive quadripole are assumed. All quantities below are complex.

Let E_1 and E_2 be the e.m.f.'s or voltage rises at the terminals of a quadripole (Fig. 1) in the directions shown and let E be the e.m.f. within the quadripole. Then, assuming mesh currents in a clockwise direction, numbering the meshes and corresponding currents from 1 to n, and calling the mesh in which E appears number 3, the circuit equations are

$$\begin{aligned}
z_{11}I_1 - z_{12}I_2 - z_{13}I_3 - \cdots - z_{1n}I_n &= E_1 \\
-z_{21}I_1 + z_{22}I_2 - z_{23}I_3 - \cdots - z_{2n}I_n &= E_2 \\
-z_{31}I_1 - z_{32}I_2 + z_{33}I_3 - \cdots - z_{3n}I_n &= E \\
-z_{41}I_1 - z_{42}I_2 - z_{43}I_3 + \cdots - z_{4n}I_n &= 0 \\
\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots & \\
\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots & \\
-z_{n1}I_1 - z_{n2}I_2 - z_{n3}I_3 - \cdots + z_{nn}I_n &= 0
\end{aligned} \tag{1}$$

where z_{kk} is the self impedance of the k mesh and $z_{jk}=z_{kj}$ is the mutual impedance of the j and k meshes. From (1),

$$\begin{aligned}
I_1 &= \frac{E_1A_{11}}{D} + \frac{E_2A_{12}}{D} + \frac{EA_{13}}{D}, \\
I_2 &= \frac{E_1A_{21}}{D} + \frac{E_2A_{22}}{D} + \frac{EA_{23}}{D},
\end{aligned} \tag{2}$$

where D is the determinant formed by the z's in (1), and A_{jk} the cofactor corresponding to the j row and k column of D. Since D is symmetrical, $A_{jk}=A_{kj}$.

Fig. 1—Quadripole.

There are five independent impedances in (2). D/A_{11} is the input impedance at the 1 terminals when $E_2=E=0$; $D/A_{12}=D/A_{21}$ is the transfer impedance between E_2 and I_1 or E_1 and I_2 (reciprocity theorem), and is measured by E_2/I_1 when $E_1=E=0$ or by E_1/I_2 when $E_2=E=0$, etc. The three transfer and two input impedances thus defined may be obtained from the impedance equations, as indicated here by

determinants and cofactors, or by measurement. Since each may be measured or calculated by assuming a certain pair of e.m.f.'s zero, the impedances are sometimes called "short-circuit" impedances.

In (1), E was tacitly assumed to be in number 3 mesh only. If E is common to two meshes, 3 and 4, for example, then the fourth equation in (1) must be equated to $-E$ (infrequently $+E$) rather than 0; if E is common to more than two meshes, further similar changes must be made. However, it is always possible to write

$$\begin{aligned} I_1 &= E_1/z_A + E/z_B + E_2/z_C \\ I_2 &= E_1/z_C + E/z_D + E_2/z_F . \end{aligned} \tag{3}$$

This follows from the superposition theorem directly or from the circuit equations of the type of (1). The input impedance at 1 terminals when $E_2 = E = O$ is z_A, the transfer impedance between E_2 and I_1 or E_1 and I_2 is z_C, etc. The five impedances in (3) can be calculated or measured.

A network which has five independent impedances is shown in Fig. 2. For it,

$$\begin{aligned} z_A &= z_a\theta/[(z_a + z_b)(z_c + z_d) + z_c z_d] \\ z_B &= \theta/z_d;\ z_C = \theta/z_c;\ z_D = -\theta/z_b \\ z_F &= z_f\theta/[(z_d + z_f)(z_c + z_b) + z_c z_b] \\ \theta &\equiv z_b z_c + z_b z_d + z_c z_d , \end{aligned} \tag{4}$$

which follow from the determination of I_1 and I_2 in the form of (3). Solving these equations for z_a, z_b, etc., in terms of the input and transfer impedances, and writing[4] the subscript of each z for that z,

$$\begin{aligned} a &= \frac{ABC}{BC - A(B + C)} \\ f &= \frac{CDF}{CD - F(D - C)} \\ b &= -\theta/D;\ c = \theta/C;\ d = \theta/B \\ \theta &\equiv BCD/(D - B - C) \end{aligned} \tag{5}$$

Equations (5) completely determine the network of Fig. 2 in terms of the short-circuit input and transfer impedances irrespective of E_1, E_2 and E. If these impedances are made equal to the corresponding short-circuit input and transfer impedances of the active quadripole represented by Fig. 1, then the network of Fig. 2 thus determined will

[4] This convention is followed throughout the remainder of the paper. The D below represents z_D, not the determinant D of (2).

be exactly equivalent to that of Fig. 1, provided E of the former is the same as E of the latter. This follows[5] since with a given E_1, E_2 and E, the input and output (vector) currents of the active quadripole will be equal to the corresponding currents of the equivalent circuit, hence the power inputs and power outputs will be the same respectively, and the power losses will be the same. Equations (5) can be made symmetrical by writing $-D'$ for D.

A special case of interest is that in which E_2, which is completely arbitrary in (3) above, represents the negative of the drop through an impedance: $E_2 = -z_r I_2$. Substituting in (3), it follows that in general (for the original quadripole or any exactly equivalent section)

$$I_1 = E_1\left(\frac{1}{A} - \frac{rF}{C^2(r+F)}\right) + E\left(\frac{1}{B} - \frac{rF}{CD(r+F)}\right) \qquad (6)$$

$$I_2 = E_1F/(F+r)C + EF/(F+r)D.$$

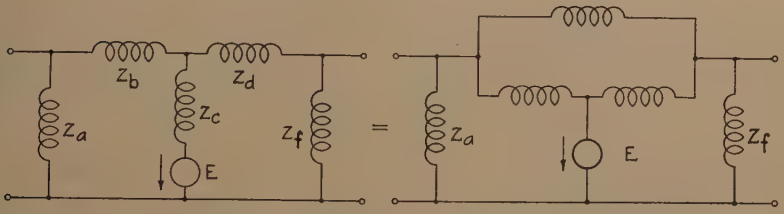

Fig. 2—Equivalent circuits (five independent impedances).

From (6) the input and transfer impedances of the quadripole with load (which of course differ from the short-circuit input and transfer impedances A, B, etc.) can be obtained:

$$\text{input impedance } (= E_1/I_1 \text{ when } E = 0) = \frac{AC^2(r+F)}{C^2(r+F) - rAF};$$

$$\text{transfer impedance between } E \text{ and } I_1 = \frac{BCD(r+F)}{CD(r+F) - BFr};$$

[5] It is practically self-evident in the physical case; from the mathematical point of view the derivation may be considered an application of the rule that if $I_1 = f_a(E, E_1, E_2)$ (original quadripole) is to equal $I_1 = f_b(E, E_1, E_2)$ (equivalent circuit) for all values of E, E_1, and E_2, then coefficients of similar terms on the two sides of the equations must be equal. The E's have been taken as the arbitrary variables for convenience here; arbitrary impedances could be used as variables in place of one or two of the e.m.f.'s as is sometimes convenient in the corresponding passive quadripole theory, or still other quantities could be considered the arbitrary variables. For example, if $-z_r I_2$ is substituted for E_2 and the equations for I_1 and I_2 for the circuit of Fig. 2 are written in the form of (6) below, then equating coefficients of E_1 and E to the corresponding coefficients in (6) *and* equating coefficients of like powers of r in the resulting equations will yield (5).

transfer impedance between E_1 and $I_2 = (F + r)C/F$;

transfer impedance between E and $I_2 = (F + r)D/F$.

The input impedance is the same as in the case of a passive quadripole.

Equations (5) may in particular cases yield impedances which have negative real parts, which are permissible mathematically but undesirable if the equivalent circuit is to be realized physically. Another circuit which may be used is shown in Fig. 3 in two forms which may

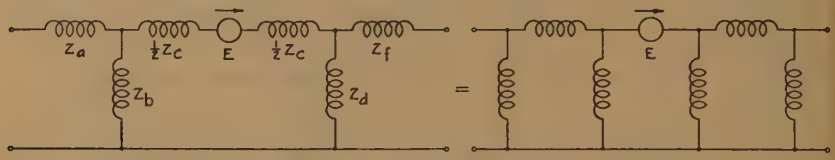

Fig. 3—Equivalent circuits (five independent impedances).

be made equivalent simply by changing the two T sections in the first form to equivalent π sections by the corresponding passive quadripole transformation. Using the first form, the conditions under which the circuit will exactly represent an active quadripole containing one e.m.f. E are

$$\begin{aligned} a &= AB(C - D)/(BC - AD) \\ b &= ABD/(BC - AD) \\ d &= BDF/(CD - BF) \\ f &= DF(C - B)/(CD - BF) \\ c &= b(B - A)/A + d(B - C)/C = d(D - F)/F + b(D - C)/C \end{aligned} \tag{7}$$

In Fig. 3 the impedance c is shown in two parts for the sake of symmetry. The division need not be made, or it can be made with any portion of c on one side of E and the remainder on the other. In particular if c-f is on the left of E and f on the right, the mesh to the right of E is a symmetrical T, hence it can be stated as a general rule that *any active quadripole containing one e.m.f., E, can be represented by two simple passive sections joined through a generator of e.m.f., E,* one of the sections being symmetrical. This follows since any passive quadripole can be represented by a T.

The equivalent circuits of Figs. 2 and 3 are the simplest possible in general, corresponding to the simple π and T sections of passive transducer theory. It has been assumed that an e.m.f., E, is used in the equivalent circuit; as shown below, the equivalent circuit may be reduced when this condition is disregarded.

By combining (7) and (4), it is possible to obtain the impedances a, b, c, d, f appearing in (7) in terms of those appearing in (4), thus formally determining the equivalent of the circuit of Fig. 2 in the form of that of Fig. 3 or vice versa. Denoting the impedances of the circuit of Fig. 3 by a subscript 1, the results are

$$\begin{aligned} a_1 &= a(b+c)/(a+b) \\ b_1 &= -ac/(a+b) \\ c_1 &= -c\left[\frac{a(c-d)+f(b+c)+c(b+d)}{(a+b)(f+d)}+1\right] \\ d_1 &= cf/(d+f) \\ f_1 &= f(d-c)/(d+f) \end{aligned} \tag{7a}$$

and,

$$\begin{aligned} a &= \theta_1/[f_1(b_1+d_1)+(c_1-b_1)(d_1+f_1)] \\ b &= \theta_1(a_1+b_1)/b_1(2a_1d_1+a_1f_1+b_1d_1) \\ c &= -\theta_1/(2a_1d_1+a_1f_1+b_1d_1) \\ d &= -\theta_1(d_1+f_1)/d_1(2a_1d_1+a_1f_1+b_1d_1) \\ f &= \theta_1/[a_1(b_1+d_1)+(c_1+d_1)(a_1+b_1)] \\ \theta_1 &\equiv [c_1(d_1+f_1)+d_1f_1](a_1+b_1)+a_1b_1(d_1+f_1). \end{aligned} \tag{7b}$$

These results permit transition from the circuit of Fig. 2 to that of Fig. 3 or vice versa, and correspond to the conversion formulas for changing one passive quadripole to another, for example a T to a π section or vice versa.

In the preceding two equivalent circuits for an active quadripole containing one e.m.f., E, it has been necessary to use five independent impedances. It is possible to set up an equivalent circuit using four independent impedances, provided an e.m.f. proportional to, but not equal to, E is used.[6] In Figs. 4 and 5 two such sections are shown. The conditions of equivalence for the circuit of Fig. 4 are

$$\begin{aligned} b &= ABC/(BC-AD) \\ c &= BC(C-F)/(CD-BF) \\ d &= BCF/(CD-BF) \\ f &= CF(D-B)/(CD-BF) \\ k &= C/D. \end{aligned} \tag{8}$$

[6] This is evident from equations (3). If E is replaced by kE then the denominator of either kE term may be made equal to some combination of A, B, C, D, F, by proper choice of k, and the number of required independent impedances in the equivalent circuit may be reduced to four. For example, the circuit of Fig. 4 may be derived from that of Fig. 3 by setting $a=0$ (7) in the latter; this requires $k=C/D$ as may be determined by replacing D by kD and B by kB in (a) and solving for k when $a=0$.

The quantity k is the factor by which D and B in (7) must be multiplied to make $a=0$ in the circuit of Fig. 3. Equations (8) may be obtained by replacing D by kD and B by kB in (7).

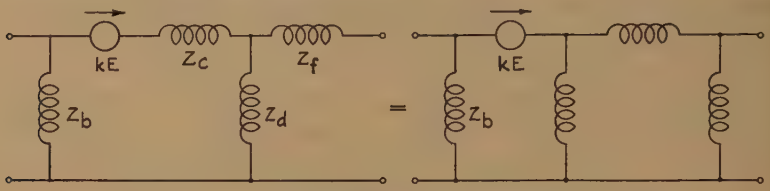

Fig. 4—Equivalent circuits (four independent impedances).

For the circuit of Fig. 5 the conditions of equivalence are

$$\begin{aligned} a &= ABCF/[BF(C-A) - AD(F-C)] \\ b &= BCF/(BF - CD) \\ c &= -Dkb/C \\ d &= -CDF/(BF - CD) \\ k &= CF/D(F-C) \end{aligned} \tag{9}$$

and k is the factor by which D in equations (5) must be multiplied to make f infinite in the circuit of Fig. 2. If D is replaced by kD and B by kB in (5), equations (9) result.[7]

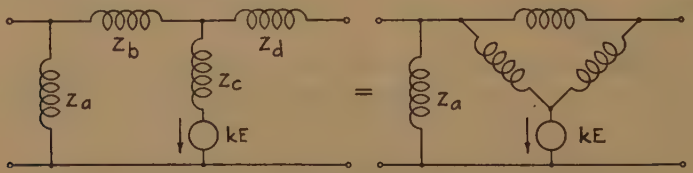

Fig. 5—Equivalent circuits (four independent impedances).

[7] For a particular network all the impedances which are pure resistances, $A=8.38$, $B=21.1$, $C=30.7$, $D=22.1$, $F=11.31$ (ohms throughout except as noted below). The equivalent circuits are as follows: circuit of Fig. 2; $a=25.3$, $b=21.8$, $c=-15.7$, $d=-22.8$, $f=9.88$; circuit of Fig. 3; $a=3.28$, $b=8.44$, $c=9.1$, $d=12.0$, $f=5.47$; circuit of Fig. 4: $b=11.7$, $c=27.2$, $d=16.7$, $f=0.79$, $k=1.39$ (pure number); circuit of Fig. 5: $a=6.9$, $b=-16.7$, $c=-9.84$, $d=17.5$, $k=-0.818$ (pure number).

It may be noted that, in general, if $E=0$, circuits equivalent to a passive transducer are obtained These are more complex than the usual T or π section. However, they are equivalent, not only as far as external reactions are concerned, but may be made to satisfy two "*internal*" conditions, the two chosen in this paper being that B and D have specified values (conditions which do not require that E differ from zero). It is thus seen that there is a field of use for these equivalent circuits in passive transducer theory. The equivalent T section (a and c series arms, b the shunt arm) of the network of the previous paragraph is $a=5.9$, $b=3.44$, $c=9.08$, and this is the equivalent T of all passive circuits with the given A, C, and F, irrespective of B and D or any other "internal" conditions.

This illustrates a method of reducing an equivalent section containing five independent impedances to one containing four, the configuration of the latter section being the same as that of the former with the exception that one impedance of the former is missing. Equations (8) and (9) can also be obtained by the more tedious method used in deriving (5) from (4).

From the above it is possible to generalize: if an active quadripole contains n independent e.m.f.'s, its equivalent circuit will contain either the n e.m.f.'s and $2n+3$ independent impedances or $n-m$ of the original e.m.f.'s, m equivalent e.m.f.'s and $2n-m+3$ independent impedances. By equivalent e.m.f. is meant one such as kE in the two preceding equivalent circuits.

A special case of the general problem of equivalence is that in which symmetry exists in the original network, such that $A=F$ and $B=-D$. Then $b=d$ and $a=f$ in the equivalent network of Fig. 2 and $a=\infty$ and $b=d$ in that of Fig. 5. The independent parameters reduce to three, just as in the case of a passive quadripole the parameters reduce from three to two when the original network is symmetrical. In this special case the equivalent circuit of Fig. 5 reduces to that of Fig. 6. Let b and c (Fig. 6) be the impedances of the equivalent T

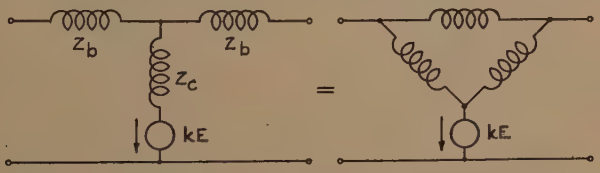

Fig. 6—Equivalent circuits of symmetrical networks.

of the original network when $E=0$, and let $k=-(2c+b)/D=(2c+b)/B$ which inspection of the figure shows to be required for E to contribute E/B to I_1 and E/D to I_2. Then

$$b = AC/(A + C); c = A^2C/(C^2 - A^2) \tag{9a}$$

and,

$$\begin{aligned} I_1 &= E_1/A + E/B + E_2/C \\ I_2 &= E_1/C + E/D + E_2/F \end{aligned} \tag{10}$$

in agreement with (3). The E_1 and E_2 terms on the right-hand side of each equation follow from the equivalent T of the original network made passive, the third term being simply added (superposition theorem) to allow for the effect of kE. The result may also be obtained directly from (9).

As another special case, consider that in which the symmetry is $A=F$ and $B=D$ in (3). The circuit of Fig. 7 is the reduction of that of Fig. 4 for this case. Let b and c be impedances of the equivalent π section of the original network without E, and let $k=c/B=c/D$. Then the equations for I_1 and I_2 are identical with (3). From equations (8), (Fig. 4),

$$b = d = AC/(C - A) \text{ and } c = C \tag{11}$$

which are the equations of transformation of any symmetrical ($A=F$) passive quadripole to a π section.

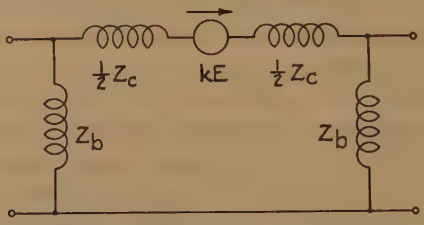

Fig. 7—Equivalent circuit of symmetrical network.

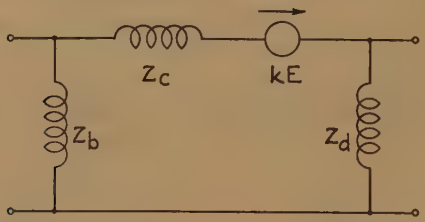

Fig. 8—Equivalent circuit for an active transducer.

As a third special case let $E=hE_1$ where h is a constant, i.e., let the e.m.f. within the quadripole be directly proportional to E_1. The circuit equations of the form of (1) will always yield equations of the form of (2) which contain *four* independent parameters:

$$\begin{aligned} I_1 &= E_1/A + E_2/G \\ I_2 &= E_1/H + E_2/F \end{aligned} \tag{12}$$

in which G is not equal to H, and all impedances except F depend on h. Equations of exactly the same form hold for a quadripole which contains *any number* of e.m.f.'s, each of which is proportional to E_1 or E_2. If all the e.m.f.'s in the quadripole are proportional to E_1, the mesh of Fig. 8 may be made to simulate it exactly. The conditions are

$$b = AH/(H - A);\ c = G;\ d = FG/(G - F);\ k = (G - H)/H.$$

A circuit of the form of that of Fig. 8 has been used for the equiva-

lent circuit of a triode amplifier. In this case $A=z_1z_2/(z_1+z_2)$; $G=z_2$; $H=z_2r_p/(r_p-\mu z_2)$ and $F=r_pz_2z_3/(r_pz_2+r_pz_3+z_2z_3)$, where z_1, z_2, z_3 are respectively the grid-filament, grid-plate, and plate-filament impedances of the tube cold. The equivalent circuit impedances and k are, assuming z_1, z_2 and z_3 large in comparison with the plate resistance r_p,

$$b = r_p/\mu;\ c = z_2;\ d = r_p;\ k = -\mu z_2/r_p.$$

A check will show that the equivalent circuit gives the usual $-I_2 = \mu E_1/(z+r_p)$ where z is the load impedance.

All the preceding equivalent circuits are among the simpler, if not the simplest, circuits which can be set up for the cases considered. It is evident from the basic requirements of an equivalent circuit that many circuits more complex than those given in this paper may be used, but it does not appear that there will be any resulting advantage except in very special cases. This corresponds to the usual use of T and π rather than more complex sections as equivalents of passive quadripoles. In a recent paper[8] R. M. Foster has given or indicated all the forms which two-, three- and four-mesh transducers may assume. From these the numerous possible corresponding forms of the equivalent circuit of an active quadripole may be obtained. In a supplementary note it is hoped to extend the work of this paper to the general case of an active quadripole containing any number of e.m.f.'s.

[8] Ronald M. Foster, "Geometrical circuits of electrical networks," Bell System Monograph B-653.

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MUTUAL IMPEDANCE OF TWO SKEW ANTENNA WIRES*

By

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Summary—*In a recent communication[1] P. S. Carter derived a formula for the mutual impedance of two antenna wires intersecting at an angle θ, as a sum of two definite integrals. It has been shown[2] that related double integrals can be reduced to sums of logarithmic integrals and their limits; in the present paper this reduction will be generalized. The mutual impedance of two antenna wires in space which do not intersect may be calculated in terms of the integrals discussed, but these integrals can be evaluated by known tables only if the wires are in the same plane. The integrals of Carter result from a suitable limiting operation; a direct evaluation is also possible and is given.*

I.

IF d is the shortest distance between two skew lines in space, the corresponding line intersects each of the skew lines at right angles; let x, y be distances along the antenna wires (1), (2), respectively, from this intersection. If c is the cosine of the angle between the two

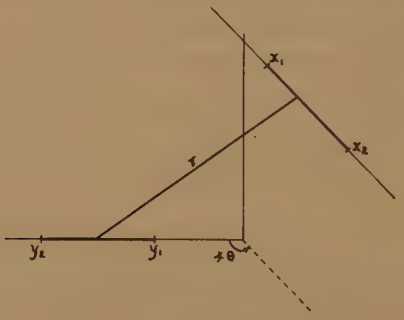

Fig. 1

skew wires, the distance between two arbitrary points, one on each wire, can be written

$$r = [d^2 + x^2 + y^2 - 2cxy]^{1/2}$$
$$= [d_y^2 + (x - cy)^2]^{1/2} = [d_x^2 + (y - cx)^2]^{1/2}$$
$$d_y^2 = d^2 + y^2(1 - c^2),\ d_x^2 = d^2 + x^2(1 - c^2).$$

* Decimal classification: R120. Original manuscript received by the Institute, July 16, 1932.

[1] "Circuit relations in radiating systems and applications to antenna problems," Proc. I. R. E., vol. 20, p. 1041; June, (1932).

[2] F. H. Murray, "On the numerical calculation of the current in an antenna," *Amer. Jour. Math.*, vol. 53, pp. 889–890, (1931).

If the current on each wire can be represented as a sum of terms representing waves which are traveling with the velocity of light, the calculation of the mutual impedance requires the evaluation of integrals

$$I = \int_{x_1}^{x_2} e^{-\alpha x} dx \int_{y_1}^{y_2} e^{-\beta y - hr} dy/r, \quad h = jk = 2\pi j/\lambda.$$

in which $\alpha = \pm jk$, $\beta = \pm jk$. These four possibilities can be reduced to the one $\alpha = jk$, $\beta = jk$, by transformations $x' = \pm x$, $y' = \pm y$, $c' = \pm c$. To simplify I, introduce a new variable t:

$$\begin{aligned} d_x t &= r + (y - cx), & r &= (d_x/2)(t + 1/t), \\ d_x/t &= r - (y - cx), & y - cx &= (d_x/2)(t - 1/t). \end{aligned} \qquad \partial y/\partial t = r/t.$$

Then,

$$\begin{aligned} I &= \int_{x_1}^{x_2} e^{-hx} dx \int_{t_1}^{t_2} e^{-h(y+r)} dt/t \\ &= \int_{x_1}^{x_2} e^{-h(1+c)x} dx \int_{t_1}^{t_2} e^{-h d_x t} dt/t = \int_{x_1}^{x_2} e^{-h(1+c)x} dx \int_{k d_x t_1}^{k d_x t_2} e^{-ju} \frac{du}{u} \\ &= -(1/h(1+c)) \left\{ \left[e^{-h(1+c)x} \int_{k d_x t_1}^{k d_x t_2} e^{-ju} du/u \right]_{x=x_1}^{x=x_2} \right. \\ &\quad \left. - \int_{x_1}^{x_2} e^{-h(1+c)x} \left[e^{-h d_x t} \partial(d_x t)/d_x t \partial x \right]_{t=t_1}^{t=t_2} dx \right\}. \end{aligned} \tag{1}$$

Now the first expression can be evaluated in terms of the *Ei* functions; in the second,

$$\begin{aligned} &(1/d_x t_i)\partial(d_x t_i)/\partial x \\ &\quad = (1/(r_i + y_i - cx))(\partial r_i/\partial x - c) \\ &\quad = (1/(r_i + y_i - cx))((x - cy_i)/[d_y^2 + (x - cy_i)^2]^{1/2} - c). \end{aligned}$$

The second expression in (1) becomes a sum of integrals

$$J_i = \int_{x_1}^{x_2} e^{-h(r+x+y_i)} (r_i + y_i - cx)^{-1} ((x - cy_i)/[d_y^2 + (x - cy_i)^2]^{1/2} - c) dx.$$

Transforming as before,

$$\begin{aligned} d_y u &= r_i + (x - cy_i), & r_i &= (d_y/2)(u + 1/u), \\ d_y/u &= r_i - (x - cy_i), & x - cy_i &= (d_y/2)(u - 1/u), \end{aligned} \qquad \partial x/\partial u = r_i/u.$$

and,

$$J_i = e^{-h y_i(1+c)} \int_{u_1}^{u_2} e^{-h d_{y_i} u}(u^2 - \gamma^2)du/u[u^2 + 2uy_i(1+c)/d_y + \gamma^2],$$

$$\gamma^2 = (1+c)/(1-c).$$

If,

$$\alpha_1 = [-y_i(1+c) + jd\gamma]/d_{y_i}, \quad \alpha_2 = [-y_i(1+c) - jd\gamma]/d_{y_i},$$

$$J_i = e^{-h y_i(1+c)} \int_{u_1}^{u_2} e^{-h d_{y_i} u}[-1/u + 1/(u-\alpha_1) + 1/(u-\alpha_2)]du.$$

If γ is finite and different from zero, and d is not zero, this integral can be expressed as a sum of Ei functions with complex argument:

$$J_i = e^{-h y_i(1+c)} \left\{ -\int_{kd_y u_1}^{kd_y u_2} e^{-jw}dw/w + e^{-h d_y \alpha_1} \int_{kd_y(u_1-\alpha_1)}^{kd_y(u_2-\alpha_1)} e^{-jw}dw/w \right.$$

$$\left. + e^{-h d_y \alpha_2} \int_{kd_y(u_1-\alpha_2)}^{kd_y(u_2-\alpha_2)} e^{-jw}dw/w \right\}.$$

Hence if,

$$F(x) = e^{-h(1+c)x} \int_{kd_x t_1}^{kd_x t_2} e^{-ju}du/u$$

$$I = -[h(1+c)]^{-1}[F(x_2) - F(x_1) + J_1|_{y=y_1} - J_2|_{y=y_2}].$$

II.

Transformations similar to those employed above may be used to simplify formula (63) of Carter.[1] This can be written

$$Z_{21} = -30lj(-1)^n \int_\epsilon^a e^{-hr}(\sin ks/rs)ds,$$

$$= -15l(-1)^n(I_1 - I_2),$$

$$k = 2\pi/\lambda, \quad r = [l^2 + s^2 - 2lsc]^{1/2},$$

$$h = jk, \qquad c = \cos\theta,$$

$$I_1 = \int_\epsilon^a e^{-h(r-s)}ds/rs,$$

$$I_2 = \int_\epsilon^a e^{-h(r+s)}ds/rs.$$

$$r^2 = l^2(1-c^2) + (s-cl)^2 = d^2 + (s-cl)^2,$$

$$d = l[1-c^2]^{1/2} = l\sin\theta.$$

[1] *Loc. cit.*, p. 1041.

To simplify the first integral, let

$$dt = r - (s - cl), \qquad r = (d/2)(t + 1/t),$$
$$d/t = r + (s - cl), \quad s - cl = (d/2)(1/t - t), \qquad \partial s/\partial t = -r/t.$$

from which,

$$I_1 = -\int_{t_1}^{t_2} e^{-h(dt-cl)} dt/t[cl - (d/2)(t - 1/t)]$$

$$= (2e^{hcl}/d) \int_{t_1}^{t_2} e^{-hdt}/(t - \alpha)(t + 1/\alpha), \quad \alpha = (1 + c)^{1/2}(1 - c)^{-1/2}.$$

Expanding in partial fractions,

$$I_1 = e^{hcl}/l\left[e^{-hd\alpha} \int_{-t_1+\alpha}^{-t_2+\alpha} e^{+jkdw} dw/w - e^{hd/\alpha} \int_{t_1+1/\alpha}^{t_2+1/\alpha} e^{-jkdw} dw/w\right]. \quad (2A)$$

The second integral is reduced similarly by

$$du = r + (s - cl), \; d/u = r - (s - cl).$$

$$I_2 = -e^{-hcl}/l\left[e^{hd\alpha} \int_{u_1+\alpha}^{u_2+\alpha} e^{-jkdw} dw/w \right.$$

$$\left. - e^{-hd/\alpha} \int_{u_1-1/\alpha}^{u_2-1/\alpha} e^{-jkdw} dw/w\right]. \quad (2B)$$

Since t_1 approaches α, u_1 approaches $1/\alpha$, and

$$h(cl - d\alpha) = -hl = -h(cl + d/\alpha),$$

the first integral of (2A) can be combined with the second of (2B) before taking the limit $\epsilon = 0$.
In terms of the functions

$$Cix = -\int_x^\infty \cos x dx/x, \quad Six = \int_0^x \sin x dx/x$$

we have,

$$D = \int_{\alpha-t_1}^{\alpha-t_2} e^{jkdw} dw/w - \int_{u_1-1/\alpha}^{u_2-1/\alpha} e^{-jkdw} dw/w$$

$$= Ci[kd(\alpha - t_2)] - Ci[kd(\alpha - t_1)] + Ci[kd(u_1 - 1/\alpha)]$$
$$- Ci[kd(u_2 - 1/\alpha)] + j\{Si[kd(\alpha - t_2)]$$
$$+ Si[kd(u_2 - 1/\alpha)] - Si[kd(\alpha - t_1)]$$
$$- Si[kd(u_1 - 1/\alpha)]\}.$$

From the definition,

$$r_1 = [l^2 + \epsilon^2 - 2l\epsilon c]^{1/2} = l - c\epsilon + \epsilon^2(\cdots),$$

$$du_1 = r_1 + \epsilon - cl, \quad dt_1 = r_1 - (\epsilon - cl),$$

$$\alpha - t_1 = \epsilon\alpha/l + \epsilon^2(\cdots), \quad u_1 - 1/\alpha = \epsilon/l\alpha + \epsilon^2(\cdots).$$

If δ is a small quantity,[3]

$$Ci\delta = 0.5772 + \log\delta + \delta^2(\cdots), \quad Si\delta = \delta(\cdots),$$

hence,

$$\lim_{\epsilon=0} D = \log[(1-c)/(1+c)] + Ci(kX) - Ci(kY) + j[Si(kX) + Si(kY)],$$

$$X = l + a - r_2, \quad Y = r_2 + a - l, \quad r_2 = [l^2 + a^2 - 2lac]^{1/2},$$

from which,

$$(e^{hl} = e^{-hl} = (-1)^n),$$

$$Z_{21} = -15(-1)^n\left\{e^{-hl}\lim_{\epsilon=0} D + e^{hl}\int_{1/\alpha+\alpha}^{u_2+\alpha} e^{-jkdw}dw/w - e^{hl}\int_{\alpha+1/\alpha}^{t_2+1/\alpha} e^{-jkdw}dw/w\right\}$$

$$= -15\left\{\lim_{\epsilon=0} D + Ci(kU) - Ci(kV) - j[Si(kU) - Si(kV)]\right\},$$

$$U = r_2 + l + a, \quad V = r_2 + l - a.$$

[3] Jahnke-Emde, "Funktionentafeln", p. 19.

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BOOK REVIEWS

Communication Engineering, by W. L. Everitt, Ohio State University. Published by McGraw-Hill Book Co., Inc., New York. 567 pages, 323 figures. Price $5.00.

This is a textbook suitable for college students in electrical engineering, for electrical engineers in general, and for readers who are familiar with the essentials of direct- and alternating-current theory. The emphasis is laid by the author upon fundamental principles, rather than upon a specialized treatment of apparatus or applications.

After an introductory chapter which affords a survey of the nature of the problems involved in electrical communication, a thorough treatment of circuits follows. About half of the book is devoted to such fundamental subjects as network theorems, resonance, the infinite line, reflection, the theory of filters, impedance, and coupled circuits.

Vacuum tube theory is very fully treated: amplifiers from the standpoint of unilateral impedances, modulation and detector action from that of nonlinear impedances, and oscillators with reference to negative resistance effect. The treatment is based on the use of equivalent simplified circuits and covers practical requirements in a satisfactory manner. Chapters on electromechanical coupling and on radiation follow, with a section on the commoner methods of medium- and high-frequency measurements in conclusion.

The method of presentation is clear, the order logical, the figures excellent. This work should prove to be a teachable college textbook, and a valuable reference book for the engineer.

[1]F. W. Grover

[1] Union College, Schenectady, N. Y.

Radio Engineering, by F. E. Terman, Stanford University. Published by McGraw-Hill Book Co., Inc.. New York. Price $5.00.

The value of a new radio book is naturally determined by the extent to which it possesses merit in two respects; namely, the effectiveness and completeness of the presentation of fundamental principles; and the amount and importance of material included which is not readily obtainable elsewhere. Professor Terman's book possesses merit in both of these respects.

As the title implies the emphasis is on the engineering aspect of the subject. The treatment is descriptive and analytical with a minimum of mathematics. Although a few errors of minor significance are observed, for the most part it is free from serious mistakes.

His analysis of the properties of resonance circuits is very good. The short chapter on the fundamental properties of vacuum tubes contains a great deal of information in condensed form. The student, and also the engineer who is interested in practical applications, will find much of value in the 108 pages devoted to amplifiers and in the forty pages on vacuum tube detectors. Both subjects are carefully analyzed and are supplemented by many experimental curves that are usually not available to the reader. Other closely related subjects of a broad

theoretical interest to which considerable space is devoted are: oscillators, special tubes, and modulation. These are followed by some of the more practical aspects of power supply, transmitters, and receivers. It is the feeling of the reviewer that the twenty photographs of equipment detract rather than add to the value of the book as they do not contribute to a better understanding of the systems.

The various types of simple and directional antennas are described and their characteristics discussed and illustrated by curves showing horizontal and vertical directivity. This is followed by an analysis of the element involved in the propagation of radio waves to which thirty pages are devoted. The last two chapters deal with high-frequency measurement and sound and sound equipment.

The book will be found stimulating and will make a valuable addition to a radio engineer's library.

[2]H. M. Turner

[2] Yale University, New Haven, Conn.

Photocells and Their Applications (Second edition), by V. K. Zworykin, and E. D. Wilson, 305 pages. Price $3. John Wiley & Sons, New York City.

The scientific literature available on photo-electric effect and photocells is most voluminous and, in addition, much has appeared in the popular technical press during the past few years. However, for the radio or electrical engineer desiring specific, applicable information, there has been a real lack of material without a time consuming search. This has been particularly true as regards fundamentals in the application of these devices.

About twelve years ago the situation was similar as regards vacuum tubes when van der Bijl's "The Thermionic Vacuum Tube" and J. H. Morecroft's "Principles of Radio Communication" appeared and met the need. It is believed that this book on photocells will gain a similar reputation in its own field.

The subject is covered broadly and thoroughly, and includes numerous useful tables, illustrations, and references.

This second edition is not only revised but contains a number of entirely new chapters. There are frequent references to and illustrations of modern devices. These add to the value of the book but will necessitate revisions of this book from time to time to enable it to keep its place in the field.

It is a book that can be recommended highly to radio and electrical engineers.

[3]W. C. White

[3] General Electric Co., Schenectady, N. Y.

Handbook of Chemistry and Physics (Seventeenth Edition), Chemical Rubber Publishing Co., Cleveland, Ohio. 1722 pages, 4¼ inches x 6½ inches. Price $6.00.

A book in the seventeenth edition should need little more than an informal introduction.

This edition of 1932 differs from the sixteenth, 1931, edition in that there are 187 more pages. Wholly new material, to the extent of about thirty-five pages, has been added. The most important revision has been that of the table "Physical Constants of Organic Compounds," which has been recompiled, thus increasing the number of pages by about one hundred. The following tables are given for

the first time: Powers of Numbers; Factorials and their Logarithms; Factors for Computing Probable Errors; Probability of Occurrence of Deviations; Areas, Ordinates, and Derivatives of the Normal Curve of Errors; Arrangements of Electrons in Orbits; Correction of Boiling Points to Standard Temperature; Index of Refraction of Aqueous Solutions of Sucrose; Vapor Pressure, Variation with Temperature; Conversion Table for Transmission Units.

For those who are not acquainted with the earlier editions it may be said that the "Handbook of Chemistry and Physics" is a compilation of mathematical formulas and tables, and of chemical and physical constants and laws. To quote from the preface, "An attempt has been made to include material on all branches of chemistry and physics and the closely allied sciences which would be likely to find extended use."

Some twenty pages are given to radio formulas and tables. This section contains formulas for calculating the capacity of spheres, cylinders, parallel wires, etc., for computing the inductance of coils of various shapes; values of LC; radio resistance of wires; characteristics of receiving vacuum tubes; and conversion tables for transmission units.

The "Handbook" should be at the elbow of every chemist, physicist, and engineer. It will be found to be a very present help in time of trouble.

[4]R. R. Ramsey

[4] Indiana University, Bloomington, Ind.

High Selectivity Tone-Corrected Receiving Circuits,* by F. M. Colebrook. Published by His Majesty's Stationery Office, London, for the Department of Scientific and Industrial Research. 69 pages, paper cover. Price 1s.3d. net.

This circular is the report of a theoretical and experimental investigation of receiving circuits having such high selectivity that the side band attenuation must be corrected in the audio amplifier. The work was mostly carried on at the National Physical Laboratory, between April, 1931, and September, 1932.

The first half of the paper is devoted to a theoretical discussion of the subject, describing the behavior of the essential parts of the receiving circuits, including either square-law or linear rectification, in the presence of two modulated signals whose carrier frequencies are slightly different. The second half is devoted to experimental procedures and results, using either a highly regenerative tuned circuit or a quartz crystal as the highly selective element of the system. The author has succeeded in making this subject easily understood without any sacrifice of accuracy or completeness.

The conclusions are entirely in accordance with the accepted side band theories. The system is found to exhibit a considerable advantage in reducing interference having the same character as the modulation of the undesired signal, but no advantage in reducing interference which results from the side bands of the undesired signal beating with the carrier of the desired signal. The conclusions are well presented in the author's summary and also in a less technical preface by Lieutenant-Colonel A. G. Lee. There is an excellent list of references on this subject.

[5]Harold A. Wheeler

* This is the subject which recently excited some controversy under the name "Stenode."

[5] Hazeltine Service Corp., Bayside, L. I., New York.

BOOKLETS, CATALOGS, AND PAMPHLETS RECEIVED

Copies of the publications listed on this page may be obtained gratis by addressing a request to the publisher.

H. W. Sullivan, Ltd., of Leo St., Peckham, S.E.15, London, England, has issued four catalogs covering, respectively, wavemeters and other radio-frequency apparatus, precision direct-current measuring instruments, precision alternating-current measuring apparatus, and laboratory condensers for all frequencies.

Bulletin No. 100 of the Rubicon Company, 29 North 6th St., Philadelphia, Pa., covers resistance standards and resistance measuring devices. Their circular No. 130 describes attenuation pads for audio-frequency use.

The Akaformer Noise Reducing System is the title of Engineering Bulletin 1A issued by Amy, Aceves, and King, Inc., 11 West 42nd St., New York, N. Y. It concerns connections between an antenna and receiver designed to eliminate inductive interference.

The Central Scientific Company of Chicago, Ill., has issued a new bulletin known as Cenco News Chats giving information on new products and developments in laboratory apparatus and supplies. Their bulletin No. 4 covers a complete list of rheostats for laboratory use.

The Bakelite Review issued by Bakelite Corporation of 247 Park Ave., New York, N. Y., covers interesting achievements in the adaptation of bakelite to various manufacturing, sales, and laboratory purposes.

The Emicon is an electronic musical instrument which may be used alone or in conjunction with a radio receiver, and is described in a booklet issued by Emicon, Inc., 2 West 46th St., New York, N. Y.

The Solar Manufacturing Company of 599 Broadway, New York, N. Y., has issued a leaflet describing their inverters which permit the operation of alternating-current equipment from direct-current supplies.

The Ohmite News, of interest to those having resistor problems, is available from the Ohmite Manufacturing Company, 636 North Albany Ave., Chicago, Ill. A new resistor chart which eliminates calculations arising from the use of resistor units in parallel is also available.

The Brush Development Company of 3715 Euclid Ave., Cleveland, Ohio, has issued some leaflets covering piezo-electric crystal loud speakers and microphones for sound picture, public address, and home receiver purposes.

Potentiometers, resistance boxes, Wheatstone bridges, and a new galvanometer are covered in some literature recently issued by the Gray Instrument Company located at 64 West Johnson St., Germantown, Philadelphia, Pa.

A leaflet announcing the availability of a new recording milliampere of the strip-chart type which may be obtained in ranges giving full-scale deflection for as low as five milliamperes has been issued by the Esterline-Angus Company of Indianapolis, Ind.

Oil-impregnated, electrolytic, paper, and mica condensers for all types of services are covered in the 1933 edition of the catalog of the Dubilier Condenser Corporation of 4377 Bronx Blvd., New York, N. Y.

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RADIO ABSTRACTS AND REFERENCES

THIS is prepared monthly by the Bureau of Standards,* and is intended to cover the more important papers of interest to the professional radio engineer which have recently appeared in periodicals, books, etc. The number at the left of each reference classifies the reference by subject, in accordance with the "Classification of Radio Subjects: An Extension of the Dewey Decimal System," Bureau of Standards Circular No. 385, obtainable from the Superintendent of Documents, Government Printing Office, Washington, D. C., for 10 cents a copy. The classification also appeared in full on pp. 1433–1456 of the August, 1930, issue of the PROCEEDINGS of the Institute of Radio Engineers.

The articles listed are not obtainable from the Government or the Institute of Radio Engineers, except when publications thereof. The various periodicals can be secured from their publishers and can be consulted at large public libraries.

R100. RADIO PRINCIPLES

R111.6 H. deBellescize. La reception synchrone. (Synchronous reception.) *L'Onde Electrique*, vol. 11, pp 209–224, May; pp. 225–240, June; pp. 241–256, July; pp. 257–272, August, (1932).

The principles and possibilities of synchronous reception are discussed.

R113.6 L. Bergmann. Die Erzeugung Zirkular Polarisierter elektrischer Wellen durch einmalige total Reflexion. (The production of circularly polarized electric waves by a single total reflection.) *Physikalische Zeit.* vol. 33, pp. 582–583; August 1, 1932.

A description is given of a lecture experiment for the production of circularly polarized electric waves by total reflection at a water air surface. 252-centimeter waves are reduced to 28-centimeter waves on passing through water to a reflector which is in the water. The reflected waves are observed by means of an antenna and glow lamp. They are found to be circularly polarized.

R114 ×R113.61 Atmospheric conditions and the Kennelly-Heaviside layer. *Nature* (London), vol. 130, pp. 627–628; October 22, (1932).

A report is given which correlates conditions of the weather with field strength of radio signals received from KDKA.

R114 M. F. Link. Enregistrements de parasites atmospheriques. (Recording atmospheric disturbances.) *Comptes Rendus*, vol. 195, pp. 619–621; October 10, (1932).

A note on observations of atmospheric disturbances recorded at the Observatoire du Pic-du-Midi at an altitude of 2860 meters.

R125 T. Walmsley. A new type of directive aerial. *Wireless Eng. and Exp. Wireless* (London), vol. 9, pp. 622–625; November, (1932).

A directive antenna is described, in which there is no reflector curtain. Radiating elements of various lengths are used in conjunction with each other. The nonradiating termination inserted at the end of the feeders in most types of receiving array for the purpose of matching the surge impedance of the array, is replaced by a radiating element which increases the efficiency of the system.

* This list compiled by Mr. A. H. Hodge and Miss E. M. Zandonini.

R140 Ingenious circuits in new radio receivers. *Electronics*, vol. 5, pp. 330–331; November, (1932).

Quiet automatic volume control and several other additions or changes to be made in 1933 radio sets are described.

R140 M. Osnos. Spannungen, Widerstände und Abstimmung bei gedämpften und ungedämpften Kreisen. (Potentials, resistances, and tuning in damped and undamped circuits.) *Hochfrequenz. und Elektroakustik*, vol. 40, pp. 103–108; September, (1932).

It is shown that the general definitions "capacitive resistance $= -1/\omega c$" and "inductive resistance $= \omega L$" are not tenable in theory. New definitions are derived which are valid. They are: "capacitive resistance $= (\cos^2 d/c) \cdot \tan \omega t$" and "inductive resistance $= -\omega L \tan \omega t$."

R142.3 A. J. Christopher. Transformer coupling circuits for high-frequency amplifiers. *Bell Sys. Tech. Jour.*, vol. 11, pp. 608–621; October, (1932).

This article deals with the use of transformer type of coupling circuits in high-frequency amplifiers to transmit efficiently voltages or currents between certain limiting frequencies while attenuating those above and below the limiting frequencies. Means of obtaining uniformly high amplification over relatively wide frequency bands are explained.

R143 C. W. Oatley. The theory of band-pass filters for radio receivers. *Wireless Eng. and Exp. Wireless* (London), vol. 9, pp. 608–614; November, (1932).

The article is divided into three parts. In the first of these the theory of the symmetrical two-stage filter is developed, and various formulas are derived. In the second part, consideration is given to the effects which may arise in actual filters due to lack of symmetry in the two halves of the filter. The effects of faulty gauging are dealt with. The third part is devoted to an experimental confirmation of the preceding theory.

R163 S. Takamura. The radio receiver characteristics related to the side-band coefficient of the resonance circuit. Proc. I.R.E., vol. 20, pp. 1774–1801; November, (1932).

In this paper a theoretical treatment of the modulated current in resonant circuits is given. A factor called the side-band coefficient is introduced and its utility in the calculation of output response, selectivity, and fidelity is illustrated. A comparison of the side-band coefficient and the resonance curves is made, and experimental data showing various applications are given.

R200. Radio Measurements and Standardization

R201.5 ×R387.1 H. Kaden. Die Schirmwirkung metallischer Hüllen gegen magnetische Wechselfelder. (The shielding action of metallic screens against alternating magnetic fields.) *Hochfrequenz. und Elektroakustik*, vol. 40, pp. 92–97; September, (1932).

The shielding action is calculated for simple screens. Thin-walled screens act, for low frequencies, as short-circuited windings, while for higher frequencies and comparatively thick walls the shielding effect involves current and skin effect. The required wall thickness for a given screening effect and given external dimensions is obtainable from curves which are given.

R201.7 K. Buss and A. Pernik. Vergleich von Elektronen- und Lichtschwärzung beim Kathodenoszillographen. (Comparison of electron and light darkening in cathode ray oscillography.) *Archiv für Elektrotech.*, vol. 26, pp. 723–724; October, (1932).

Electron and light blackening were compared photometrically. The ratio was found to be approximately 2.5 to 1.

R207 A. T Starr. A note on impedance measurement. *Wireless Eng. and Exp. Wireless* (London), vol. 9, pp. 615–617; November, (1932).

A method of measuring impedance is described. The use of inductive ratio arms in a Wheatstone bridge is explained.

R214 v.Petrzílka. Turmalinresonatoren bei kurzen und ultrakurzen Wellen. (Tourmaline resonators at short and ultra-short waves.) *Ann. der Physik*, vol. 15, pp. 72–87 (1932).

It is shown that it is difficult to use tourmaline as a light resonator at high frequencies. By a parallel circuit arrangement one is able to use tourmaline resonators at high frequencies. The resonant point determines the frequency to 0.02 per cent. A method for using the energy of an oscillating tourmaline plate is given.

R214 Eine neuartige Methode der Temperaturkontrolle von Quarzkristallen. (A new method of temperature control of quartz crystals.) *Elektrotech. Zeit.*, vol. 53, p. 1039; October 27, (1932).

It is suggested that the quartz crystal be mounted in a holder to which is attached a thermocouple. This method of control would be directly sensitive to heat produced by the crystal in vibration. The thermocouple of course actuates a meter.

R241 P. B. Taylor. Method for measurement of high resistance at high frequency. Proc. I.R.E., vol. 20, pp. 1802–1806; November, (1932).

A simple method is described for accurately measuring high resistance at radio frequencies in terms of a capacity and a small resistance of known value.

R262 W. N. Tuttle. Dynamic tube measurements over wide ranges of values. *Electronics*, vol. 5, pp. 344–345; November, (1932).

Data are given illustrating extensions of the use of dynamic methods of vacuum tube measurements. The data were obtained on the General Radio type 561-A vacuum tube bridge.

R262.5 J. H. Potts. Building a direct-reading mutual conductance meter. *RadioCraft*, vol. 4, p. 341; December, (1932).

This article describes a method of constructing a tube tester that reads mutual conductance directly.

R262.8 J. R. Nelson. Calculation of output and distortion in symmetrical output systems. Proc. I.R.E., vol. 20, pp. 1763–1773; November, (1932).

The conventional formula for power output is accurate when even harmonics only are present but it does not give the true power output if odd harmonics are present. It is shown that the correction factor to be applied is the square of one plus or minus the ratio of the third-harmonic amplitude to that of the fundamental. Output systems are discussed in which the even harmonics are inherently low, such as those employing single pentodes or two triodes in series, either as class A or class B amplifiers. Methods of computing the power output and distortion for each case are given together with checks between the computed and measured values. An auxiliary diagram is constructed for class A operation so that the three different output systems may be treated alike. A discussion of the point where the change from class A to B theory should be made is given for tubes connected in series.

R264.2 A. J. Palermo. The effect of displacement currents on the high-frequency resistance of circular single-layer coils. Proc. I.R.E., vol. 20, pp. 1807–1810; November, (1932).

In existing theoretical formulas for obtaining the high-frequency resistance of circular single-layer coils, the effect of displacement currents has been neglected. It was thought that the displacement currents in going from one turn of the coil to another through the dielectric, would affect the distribution of current over the cross section of the conductor and, consequently, the effective resistance at high frequencies. The present paper shows that this effect is of minor importance by applying, for the first time, the similitude principle to circular single-layer coils. The paper also experimentally substantiates the results predicted by the similitude principle.

R265.2 N. W. McLachlan. Methods of investigating the vibrational frequencies of conical shells and loudspeaker diaphragms. *Wireless Eng. and Exp. Wireless* (London), vol. 9, pp. 626–628; November, (1932).

The applications and limitations of known methods to ascertain the symmetrical vibrational frequencies of conical shells are described. The first method involves bridge measurement of motional resistance and inductance; the second involves measurement of the steady air pressure on the axis; while the third consists in recording the acoustic output when the shell is electrically impulsed.

R300. Radio Apparatus and Equipment

R330 L. Martin. New tube announcements. *RadioCraft*, vol. 4, pp. 334–335; December, (1932).

The following new tubes are described: 48, 262-A, 842, and the 59. Characteristic curves are given.

R331 Anti-microphonic valves. *Jour. Sci. Instr.*, vol. 9, pp. 325–327; October, (1932).

Description of the method of construction used by the Mullard Wireless Service Co., London, for the elimination of microphonic disturbances.

R355.7 C. E. Denton. How to make a battery-operated portable P. A. system. *RadioCraft*, vol. 4, pp. 330–331; December, (1932).

Complete constructional details of a battery-operated, portable public address system that is simple, economical, and entirely self-contained.

R355.9 ×R355.65 K. A. MacKinnon. Crystal control applied to the dynatron oscillator. Proc. I.R.E., vol. 20, pp. 1689–1714; November, (1932).

This paper describes the results of a two-year research which comprised the development of several controlled dynatron oscillators and the experimental determination of frequency stability curves of these as well as of the three Pierce circuits.

R357 E. L. C. White. The screen-grid valve as frequency changer in the superhet. *Wireless Eng. and Exp. Wireless* (London), vol. 9, pp. 618–621; November, (1932).

A theoretical explanation of the operation of the screen-grid tube as a frequency changer in the superheterodyne circuit is given.

R363 W. O. Schumann. Über Selbsterregung von Verstärkern durch Kopplung der Anodenströme. (On self excitation of amplifiers by coupling the anode currents.) *Archiv für Elektrotech.* vol. 26, pp. 580–586; August 3, (1932).

The following conclusions are drawn: In a two-tube resistance amplifier no self-exciting back coupling of the two anode currents is possible. In a two-tube transformer amplifier self-excitation is possible with ohmic and capacitative back coupling of the two anode currents. In a three-tube resistance amplifier self-excitation is possible by ohmic or by capacitative coupling between the anode currents of the first and third tube.

R385.5 J. Weinberger. The ribbon microphone and its applications. *Electronics*, vol. 5, pp. 336–337; November, (1932).

The principle of operation of the ribbon microphone are briefly discussed. The advantages of the ribbon over the condenser microphone are pointed out.

R386 H. B. Dent. Flexible band-pass unit. *Wireless World*, vol. 31, pp. 358–359; October 14, (1932).

Construction of a simple two-range antenna filter with variable selectivity control.

R388 H. E. Hollmann. Die Braunsche Röhre bei sehr hohen Frequenzen. (The cathode ray tube at very high frequencies.) *Hochfrequenz. und Elektroakustik*, vol. 40, pp. 97–103; September, (1932).

For the case where periodic time of the deflecting potential is of the order of the electron path time between the deflecting plates, the electron motion in the periodically changing accelerating field is examined. The optimum length of tube plate is determined. Various ways of insuring plate correctness are given.

R388 H. Graupner. Über einige Versuche zur elektrostatischen Konzentrierung von Kathodenstrahlen. (On an investigation of electrostatic focusing of cathode rays.) *Archiv für Elektrotech.*, vol. 26, pp. 725–730; October, (1932).

The disadvantages of the magnetic focusing are pointed out. A scheme is given for electrostatic focusing.

R388 E. Trümper. Beitrag zur Kerroszillographie. (Contribution to Kerr-cell oscillography.) *Archiv für Elektrotech.*, vol. 26, pp. 562–659; August 3, (1932).

This paper contains an investigation of the use of the electroöptical Kerr effect for oscillographic purposes.

R390 C. H. West. Building a remote-control tuning system. *RadioCraft*, vol. 4, pp. 336–337; December, (1932).

Constructional data of a new remote-control tuning system that may be built at home. It tunes, controls volume, and shuts the set "on" and "off."

R500. Applications of Radio

R520 F. Eisner. Über Langwellen-Flugfunkverbindungen. (Long-wave communication for aircraft.) *Elektrotech. Zeit.*, vol. 53, pp. 834–838, September 1; pp. 864–866, September 15, (1932).

The power input and radiating conditions in the aircraft; shapes, values of resistance and effective heights, etc., of aerials; receiving conditions on aircraft, types of interference; calculation of ranges, are discussed.

R526.2 J. E. Miller. Radio guidance. Proc. I.R.E., vol. 20, pp. 1752–1762); November, (1932).

The writer proposes a system for radio guidance by means of the rotating radio beacon. The system described would employ two rotating radio beacons transmitting simultaneously and on the same frequency. A radio receiver carried aboard the craft to be guided would receive the combined transmission from the beacons. Special equipment for this receiver's output would take bearings from the two beacons and graphically triangulate to fix the position of the craft. Triangulation would be made, and the position of the craft shown by means of intersecting light beams thrown upon the undersurface of a map.

R555 Frequency standards—Programme of transmissions from the National Physical Laboratory. *Electrician* (London), vol. 109, p. 482; October 14, (1932). *Wireless World*, vol. 31, p. 397; October 28, (1932).

The transmission of standard frequency from the National Physical Laboratory consists of a standard frequency signal of 1000 cycles imposed on or modulating a radio-frequency carrier wave. The transmission occurs on the second Tuesday of each month in the form of a modulated wave of wavelength of 830 meters, the modulation being derived from a continuously running standard. The accuracy of the standard is about 2 parts in 10 million. The transmission schedule and procedure are given.

R583 ×R524 H. R. Lubcke. Television image reception in an airplane. Proc. I.R.E., vol. 20, pp. 1732–1740; November, (1932).

The reception of a self-synchronized cathode ray television image under conditions of far greater rigor than are to be met in practice. Employing the ultra-high frequency

of 44,500 kilocycles or 6¾ meters, images were received from the Don Lee television station W6XAO while traveling at the speed of 120 miles per hour above the city of Los Angeles.

R594 K. C. DeWalt. A study of high-frequency heating. *Electronics*, vol. 5, pp. 338–340; November, (1932).

The problem of high-frequency heating with special emphasis on the production of artificial fever in man is treated, first from a purely theoretical electrical standpoint. Second, applications and experimental results are given.

R800. Nonradio Subjects

534.3 ×R210 E. Normann. A precision tuning fork frequency standard. Proc. I.R.E., vol. 20, pp. 1715–1731; November, (1932).

The use of tuning forks as audio-frequency standard is treated. Two frequency standards employing alloy forks and one employing a steel fork were checked against each other by apparatus allowing a continuous and simultaneous checking of all three forks. Suitable circuits and requirements for high precision and constancy of output are outlined. One of the frequency standards and a method of obtaining a high power output is described.

535.38 Photoelectric control in the printing arts. *Electronics*, vol. 5, pp. 334–335; November, (1932).

Sixteen applications of photocells to printing are listed. Half-tone pictures are reproduced.

535.38 F. Waibel and W. Schottky. Einige neue Feststellungen über den Sperrschicht-Photoeffekt. (Some new observations on the Barrier-layer photo-electric effect.) *Physikalische Zeit.*, vol. 33, pp. 583–585; August 1, (1932).

Copper-oxide plates without Barrier layer show less than ten thousandth part of the spontaneous photo-electric effect of plates with a Barrier layer. The Barrier layer current which is independent of the voltage vanishes when non-Barrier boundary surfaces are used. It is concluded that forced Barrier-layer photo-electric currents exist of the order of magnitude 100–1000 times the quantum equivalent.

535.38 Sur la résponse d'une cellule photo-électrique à remplissage gazeux à un éclairement brusque. (The response of a gas-filled photo cell to a sudden illumination.) *Comptes Rendus*, vol. 195, pp. 378–380; August 1, (1932).

An experimental investigation of the lag in cells with cathodes of potassium, sensitized potassium, and caesium, and with fillings of neon, argon, and helium.

535.38 F. von Orbán. Schroteffekt und Wärmegeräusch im Photozellen Verstärker. (Shot effect and thermal noise in the photocell amplifier.) *Zeit. für tech. Physik*, vol. 13, no. 9; pp. 420–424, (1932).

The writer describes his researches on three typical photocells.

537.65 ×R214 D. W. Dye. The modes of vibration of quartz piezo-electric plates as revealed by an interferometer. *Proc. Royal Soc.* (London), vol. 138, pp. 1–16; October, (1932).

Fringe patterns obtained by interference of continuous light reflected by the surface of the vibrating quartz plate and by a stationary plane surface are examined. The interferometer is also used with intermittent light for stroboscopic examination. This reveals that the motion of the vibrating plate is nonuniform not only in amplitude but also in phase, which may differ by as much as 180 degrees between various parts of the surface.

537.65 ×R214 R. E. Gibbs. The temperature variation of the frequency of piezo-electric oscillations of quartz. *Phil. Mag.* (London), vol. 14, pp. 682–693; October, (1932).

The temperature coefficients of the frequency of piezo-electric vibrations of quartz are measured over a range of several hundred degrees for two types of vibrations, viz., (a) longitudinal vibrations along X-axis for an "X-cut," (b) shear vibrations about the Z-axis for a "Y-cut." In both cases a simple hyperbolic relation is found to connect the temperature coefficient with the temperature.

537.65
×R214

A. Székely. Eine einfache Methode zur Bestimmung des ersten Piezo Modulus von Quarz aus Messungen an Quarz-resonator. (A simple method for determination of the first piezo modulus of quartz from measurements on quartz resonator.) *Zeit. für Physik*, vol. 78, pp. 560–566; October 12, (1932).

There occurs in a quartz resonator whose frequency of oscillation is near the characteristic frequency of the quartz plate, a change of capacity and of the conductivity of the condenser. The amount of this change can be determined through a simple evaluation of the resonance curve of the oscillator. This makes possible the measurement of the first piezo modulus and the damping of the quartz plate.

538.11
×R355.9

G. R. Kilgore. Magnetostatic oscillators for generation of ultra-short waves. PROC. I.R.E., vol. 20, pp. 1741–1751; November, (1932).

An electronic oscillator of the magnetostatic type, for generating wavelengths of less than 50 centimeters is described. The operating characteristics of this oscillator are given, with particular reference to obtaining maximum output and efficiency. A thorough experimental investigation of the effect of inclining the magnetic field is presented, and its importance in obtaining maximum output is discussed. Experimental curves showing the effect of various factors on frequency are given. Measurements of power output and efficiency are described, and the results compared to a Barkhausen-Kurz oscillator.

621.313.7

H. Böhm. Betriebseigenschaften von Kupferoxydul-Trockengleichrichtern. (The working characteristics of copper-oxide rectifiers.) *Elektrotech. Zeit.*, vol. 53, pp. 1052–1054; November 3, (1932).

A copper-oxide rectifier with 1500 hours continuous operation life is studied, and its working characteristics described. A special circuit is mentioned for automatic control of continuous charging.

621.319.2

J. W. Arnold and R. C. Taylor. Linearly tapered loaded transmission lines. PROC. I.R.E., vol. 20, pp. 1811–1817; November, (1932).

Working formulas for the calculation of input impedances and attenuation are obtained for a transmission line in which the inductance and resistance per unit length are linear functions of distance, and in which the capacitance and leakance are constant. Generalized functions used in the formulas, which were developed in an earlier paper, are here expressed in terms of Bessel's functions of the initial constants and the rate of taper, a form more readily applied in some cases. For the tapered loaded submarine cable the formulas are further simplified through the use of the hyperbolic functions applicable to the smooth uniform line, with appropriate correction factors.

621.385.97

M. Kluge. Frequenzgang und Plattenbeanspruchung von Tonabnehmern. (Frequency characteristics and record wear of gramophone pick-ups.) *Hochfrequenz. und Elektroakustik*, vol. 40, pp. 55–65; August, (1932).

Difficulties involved in deriving the electrical equivalent circuit of the tone pick-up are pointed out. By numerous measurements the scheme of a typical pick-up is found and represented electrically. Measurements on the representation obtained check well with measurements on an actual pick-up.

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